

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Midterm Exam

Abstract Algebra 1

11-05-2022

Incomplete solution will not receive full mark.

1. (6 points) Let $G = \{x \in \mathbb{R} \mid x \neq 1\}$ and the binary operation for all $a, b \in G$

$$a \star b = ab - a - b + 2$$

Prove that G is a group.

2. (4 points) Let $H = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$. Prove that H is a subgroup of \mathbb{R} .
3. (5 points) Use contradiction to prove that the group $U_9 \times \mathbb{Z}_9$ is not cyclic.
4. (5 points) Prove the theorem: Every subgroup of a cyclic group is cyclic.
5. (5 points) Prove the group $G = \mathbb{Z}_3 \times U_{10}$ is cyclic or not cyclic. If cyclic, find all the generators.
6. (5 points) Find all the cosets for the subgroup $H = \langle 7 \rangle$ in the group $G = U_{25}$ and determine $[G : H]$.