

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Exam 2

Abstract Algebra 2

06–05–2008

There are 6 problems; you choose 4, no more no less.

1. Let  $f \in F[x]$  be irreducible.
  - (a) What is the meaning of  $F[x]$ ?
  - (b) What is the meaning that  $f$  is irreducible?
  - (c) Prove that if  $f \mid gh$  then  $f \mid g$  or  $f \mid h$ .
2. Let  $f \in F[x]$ .
  - (a) What is the meaning of the factor ring  $F[x]/(f)$ ?
  - (b) Prove that if  $f$  is reducible then  $F[x]/(f)$  is not an integral domain.
  - (c) Prove that if  $f$  is irreducible then every nonzero element of  $F[x]/(f)$  has an inverse.
3. Let  $Q$  be the field of rational numbers.
  - (a) What is the meaning of a minimal polynomial of  $a \in R$  over  $Q$ ?
  - (b) Find the minimal polynomial of  $\sqrt{3} + \sqrt{7}$  over  $Q$ .
  - (c) Is it true that  $Q(\sqrt{3}, \sqrt{7}) = Q(\sqrt{3} + \sqrt{7})$ ? Why or why not?
4. Assume that  $\pi$  is transcendental over  $Q$ .
  - (a) What is the meaning of a transcendental element over  $Q$ ?
  - (b) Prove that  $\pi^3$  is also transcendental over  $Q$ .
  - (c) Prove that  $\sqrt{\pi}$  is also transcendental over  $Q$ .
5. Let  $K$  be an extension field over  $F$ .
  - (a) What is the meaning of an extension field?
  - (b) What is the meaning of a simple extension?
  - (c) Prove that if  $[K : F]$  is prime then  $K$  is a simple extension.
6. Let  $K$  be an extension field over  $F$ .
  - (a) What is the meaning of an algebraic element  $a \in K$  over  $F$ ?
  - (b) What is the meaning of an algebraic extension?
  - (c) Prove that if  $[K : F]$  is finite then  $K$  is an algebraic extension.