Philadelphia University
Department of Basic Sciences

Exam 2 **Abstract Algebra 2** **02–05–2011**

Choose any 3 problems from the following 5 problems.

1. Let R be a ring.
   (a) Prove that $S = \{f \in R[x] \mid f = 0 \text{ or } \deg f = 0\}$ is a subring of $R[x]$.
   (b) Prove that S is not an ideal.
   (c) Prove that $T = \{f \in R[x] \mid \deg f \leq 1\}$ is not a subring of $R[x]$.
2. Let F be a field, so the ring $F[x]$ is commutative with unity.
   (a) Prove that $F[x]$ is an integral domain.
   (b) Prove that $F[x]$ is a principal ideal domain.
   (c) Prove that $F[x]$ is not a field.
3. Let F be a field and $f \in F[x]$.
   (a) Write the definitions of (f) and $F[x]/(f)$.
   (b) If f is reducible, prove that $F[x]/(f)$ is not a field.
   (c) If f is irreducible, prove that $F[x]/(f)$ is a field.
4. Remember that $\mathbb{Z}_n$ is a field when n is a prime.
   (a) Write the definition of an irreducible polynomial.
   (b) Prove that $x^3 - 5$ is irreducible in $\mathbb{Z}_7[x]$.
   (c) Prove that $x^2 - 2$ is reducible in $\mathbb{Z}_{17}[x]$ and factor it.
5. Suppose that $a \in \mathbb{R}$, an extension over $\mathbb{Q}$.
   (a) Write the definition of the minimal polynomial f of a over $\mathbb{Q}$.
   (b) Find f if $a = \sqrt{2} + \sqrt{7}$.
   (c) Find f if $a = \sqrt{3 + \sqrt{5}}$.

–Amin Witno