# Philadelphia University

## Department of Basic Sciences


**Exam 2** | **Abstract Algebra 2** | **08–01–2017**

1. Let $F$ be a field. Prove that the polynomial ring $F[x]$ is a principal ideal domain.
2. Let $f = 21x^4 - 60$ and $g = 8x^3 + x + 27$. Prove that $f$ and $g$ are irreducible in $\mathbb{Q}[x]$.
3. Let $f = 6x^5 + 2x^3 + 2x^2 + 3$ and $g = 4x^4 + 5$. Evaluate $\gcd(f, g)$ in $\mathbb{Z}_7[x]$.
4. Let $F$ be a field and let $f, g, h \in F[x]$ such that $\gcd(f, g) = 1$. Prove that if $f \mid h$ and $g \mid h$, then $fg \mid h$.
5. Let $\theta : \mathbb{Z}_{20} \to \mathbb{Z}_{20}$ such that $\theta(n) = 5n$ for all $n \in \mathbb{Z}_{20}$. (a) Prove that $\theta$ is a ring homomorphism, but not an isomorphism. (b) Find the kernel and the range of this homomorphism.

–Amin Witno