Midterm Exam **Abstract Algebra 2** **12/12/2022**

Choose 5 questions and write complete solution.

1. Prove that S is a subring of $M(2, \mathbb{R})$, where
$$S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$$

2. Prove that S is a subfield of $\mathbb{R}$, where
$$S = \{x + y\sqrt{5} \mid x, y \in \mathbb{Q}\}$$

3. Let R be a finite integral domain. Prove that R is a field.

4. Let R be a commutative ring and $c \in R$. Let I be an ideal of R, and let
$$S = \{x \in R \mid xc \in I\}$$
Prove that S is an ideal of R.

5. Let $R = \mathbb{Z}_3 \times \mathbb{Z}_4$ with principal ideal $I = ((0, 2))$.
   (a) Construct the multiplication table for the factor ring R/I.
   (b) Find all the units and zero divisors in R/I.

6. Let $R = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ and $S = \left\{ \begin{pmatrix} x & 2y \\ y & x \end{pmatrix} \;\middle|\; x, y \in \mathbb{Z} \right\}$. (Given that R is a subring of $\mathbb{R}$ and S is a subring of $M(2, \mathbb{Z})$.) Let $\theta : R \to S$ such that
$$\theta(a + b\sqrt{2}) = \begin{pmatrix} a & 2b \\ b & a \end{pmatrix}$$
Prove that θ is a ring isomorphism.

–Amin Witno

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