Final Exam **Abstract Algebra 2** **06/02/2023**

1. (6 points) Let $R = \{x+y\sqrt{2} \in \mathbb{R} \mid x, y \in \mathbb{Z}\}$, $S = \left\{ \begin{bmatrix} x & 2y \\ y & x \end{bmatrix} \in M(2,\mathbb{R}) \middle| x, y \in \mathbb{Z} \right\}$. Prove that R is isomorphic to S as rings.
2. (6 points) Let F be a field with $|F| = 32$. Let $S = \{x^4 \mid x \in F\}$. Prove that $S = F$.
3. (5 points) Prove that $f = 15x^3 + 2x + 1 \in \mathbb{Q}[x]$ is irreducible.
4. (6 points) Let $f = x^4 + x^2 - 1 \in \mathbb{Z}_5[x]$. Prove there exist or not exist multiple zeros of f in some extension field.
5. (5 points) Let the field $F = \mathbb{Z}_2[x]/(f)$, where $f = x^3 - x + 1$ is irreducible. Compute a^2 for all $a \in F$.
6. (6 points) Let $S = \{f(x) \in \mathbb{R}[x] \mid f'(0) = 0\}$
   (a) Prove that S is a subring of $\mathbb{R}[x]$.
   (b) Prove that S is not an ideal using counter-example.
7. (6 points) Let F be a field and $f, g, h \in F[x]$ such that $\gcd(f, g) = 1$. Prove that if $f \mid h$ and $g \mid h$, then $fg \mid h$.
8. (Bonus 4 points) Find an example of a ring R and $f \in R[x]$ such that the number of zeros of $f > \deg f$.

–Amin Witno

(Univeristy ID Number) (Name of Student)