Midterm Exam **Complex Analysis** **02/05/2023**

1. (3 points) Find all solutions $z \in \mathbb{C}$ such that $2iz^2 + z - 5i = 0$

2. (2 points) Evaluate and simplify the Principal power $(1 - i)^{4i}$

3. (3 points) Evaluate and simplify $(-1 + i\sqrt{3})^8$

4. (2 points) Evaluate and simplify $(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8})^{14}$ using de Moivre formula

5. (2 points) Let $f(z) = \cos(z^2)$. Find $u(x,y)$ and $v(x,y)$ such that $f(z) = u + iv$

6. (3 points) Find the center and radius of the circle given by $|3z + 5 + 4iz - 5i| = 10$

7. (3 points) Prove that $\cosh^2 z - \sinh^2 z = 1$ for all $z \in \mathbb{C}$

8. (3 points) Prove using the definition of limit: $\lim_{z \to 2-i} 5z + 2iz = 12 - i$

9. (3 points) Prove the limit of infinity: $\lim_{z \to \infty} \frac{z^3 + i}{5z^2 - z} = \infty$

10. (6 points) Let $f(z) = 3x^2 - y^3 + ix + 2ixy - iy^2$. Find $f'(z)$ and find the domain where it exists

11. (Bonus 2 points) Let $f(z) = e^{\bar{z}}$. Prove that $f'(z)$ does not exist for all $z \in \mathbb{C}$

12. (Bonus 2 points) Prove the formula $|\sin z|^2 = \sin^2 x + \sinh^2 y$ for all $z = x + yi \in \mathbb{C}$

–Amin Witno