Final Exam **Complex Analysis** **11/06/2023**

Each problem is worth 5 points. Final answers must be in simplest form.

1. Let $f(z) = f(x + yi) = e^x y^2 + ie^x y - 3ie^x$. Use Cauchy-Riemann equations to find the domain where $f'(z)$ exists, then evaluate $f'(z)$.

2. Let $f(z) = (\text{Log } z)^2$. Use Cauchy-Riemann equations in polar form to find the domain where $f'(z)$ exists, then evaluate $f'(z)$.

3. Prove that $u(x, y) = y^3 - 3yx^2 + 2y$ is harmonic for all $x, y \in \mathbb{R}$, then find a harmonic conjugate v such that $f(z) = u + iv$ is entire.

4. Evaluate $\int_C (z + 2)^3 \, dz$, where C is the semi-circle $z(t) = e^{it}$ $(-\frac{\pi}{2} \leq t \leq \frac{\pi}{2})$.

5. Evaluate $\int_C (\bar{z})^2 \, dz$, where C is the straight line from $-1 + 2i$ to $1 - 2i$.

6. Evaluate using Cauchy Integral Formula, where C is the circle with center at $z = 0$ and radius $R = 8$.
$$\int_C \frac{z + 1}{z^2 + 9} \, dz$$

7. Evaluate using the general form of Cauchy Integral Formula, where C is the circle with center at $z = i$ and radius $R = \frac{1}{2}$.
$$\int_C \frac{z - 1}{(z^2 - iz)^3} \, dz$$

8. Evaluate the real integral using Cauchy Integral Formula.
$$\int_0^\infty \frac{dx}{(x^2 + 1)(x^2 + 4)}$$

9. (BONUS) Evaluate the real integral using Cauchy Integral Formula.
$$\int_{-\pi}^{\pi} \frac{dx}{5 - 4 \sin x}$$

–Amin Witno