PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Discrete Structures	(210104)	Paper:	Final Exam (A)
Discrete Mathematics	(210242)	Date:	13 June 2006
Discrete Mathematics	(250151)	Time:	11:30 – 13:30

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Part One **Each problem is worth 2 points.**

1. Which proposition is a contingency?
   (a) $p \rightarrow \neg p$ (b) $p \oplus \neg p$ (c) $\neg p \leftrightarrow p$ (d) $\neg p \vee p$

2. Which equivalence is not correct?
   (a) $p \rightarrow q \equiv \neg p \vee q$ (b) $q \rightarrow \neg p \equiv p \rightarrow \neg q$
   (c) $p \oplus q \equiv \neg p \leftrightarrow q$ (d) $p \oplus \neg q \equiv \neg p \leftrightarrow q$

3. Convert the proposition $(\neg p \wedge q) \vee (\neg p \wedge \neg q)$ to a CNF.
   (a) $(\neg p \vee q) \wedge (\neg p \vee \neg q)$ (b) $(\neg p \vee \neg q) \wedge (p \vee \neg q)$
   (c) $(p \vee \neg q) \wedge (p \vee q)$ (d) $(p \vee q) \wedge (\neg p \vee \neg q)$

4. Let $P(x,y) : x^2 - y \geq 0$. Which proposition is false?
   (a) $\exists x \forall y\, P$ (b) $\exists y \forall x\, P$ (c) $\forall x \exists y\, P$ (d) $\forall y \exists x\, P$

5. Convert the binary number 101010111101 to hexadecimal.
   (a) ABE (b) BDA (c) BEA (d) ABD

6. If $|A| = 2$ and $|B| = 4$ then $|P(A \times B)| =$
   (a) 16 (b) 64 (c) 32 (d) 256

7. Convert the decimal number 995 to hexadecimal.
   (a) 3DA (b) 3E3 (c) 3D6 (d) 3E4

8. Evaluate GCD (236, 326).
   (a) 3 (b) 18 (c) 2 (d) 1

9. Find an explicit formula for the following recurrence sequence.
   $f(0) = 1$
   $f(1) = 2$
   $f(n) = 4 f(n-1) - 4 f(n-2)$ for $n \geq 2$

   (a) $f(n) = 2^n$ (b) $f(n) = 2^n + n(2^n)$
   (c) $f(n) = n(2^n)$ (d) $f(n) = \frac{1}{2}(2^n) + 2n(2^n)$

10. How many positive integers ≤ 500 which are not multiples of 20 or 30?
    (a) 476 (b) 477 (c) 467 (d) 480

11. There are 6 chapters in Discrete Structures notes. How many questions at least must be in the Final Exam to make sure ≥ 7 questions come from the same chapter?
    (a) 37 (b) 43 (c) 49 (d) 46

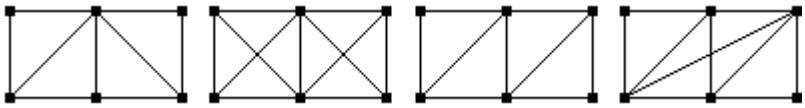
12. Which matrix represents a partial order relation?

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

13. What is the degree of the graph with degree sequence (1, 0, 2, 4, 2, 0, 1)?
(a) 28 (b) 34 (c) 30 (d) 32

14. Which degree sequence is not valid?
(a) (1,2,1,2) (b) (0,0,2,0,1)
(c) (1,1,1,1,1,1) (d) (1,0,0,1,0,1)

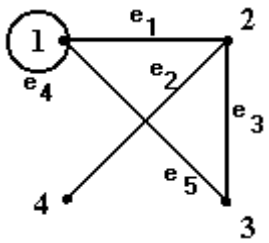
15. Which graph is an Euler path but not circuit?
(a) (b) (c) (d)



16. Convert the incidence matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

(a) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$

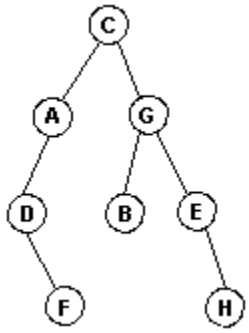
17. Which one is an incidence matrix for the following graph?



(a) $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

18. Apply the post-order algorithm for this labeled binary tree.
(a) F,D,A,C,B,H,E,G (b) F,H,D,B,E,A,G,C
(c) F,D,A,B,H,E,G,C (d) F,D,A,H,E,B,G,C



19. Apply the in-order algorithm for the same tree.
(a) D,F,A,C,B,G,H,E (b) F,D,A,C,B,G,E,H
(c) A,D,F,C,B,G,E,H (d) D,F,A,C,B,G,E,H

Part Two Each problem is worth 4 points

1. Is the following argument valid or not valid? Prove it.
Premise 1: Today is not Sunday
Premise 2: Today is Sunday if and only if tomorrow is not Friday
Conclusion: Tomorrow is Friday

2. How many positive integers ≤ 500 are not multiples of 8 or 12 or 20?

3. Represent the following expression using a labeled binary tree according to the in-order traversal algorithm.

$$\{ p \rightarrow (\neg p \leftrightarrow q) \} \rightarrow (\neg q \leftrightarrow p) \leftrightarrow q$$