Department of Basic Sciences — Philadelphia University

Final Exam **Discrete Structures** **31–01–2016**

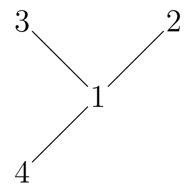
Part I. (2 points each) Circle one answer from the multiple choice.

1. Compute $\{1,3,4,6\} \oplus \{3,4,5,6\}$.
   (A) $\{1,5\}$ (B) $\{3,4\}$ (C) $\{1,5,6\}$ (D) $\{6\}$

2. Find the function that gives the sequence 2, 5, 8, 11, 14, 17, ...
   (A) n^2+2 (B) 2^n+1 (C) $3n+2$ (D) $(n+2)^2$

3. Find the matrix that represents the relation $R = \{(a,b) \mid a \bmod 2 \neq b \bmod 3\}$?
   (A) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

4. If $A = \{1,2,3,4\}$, find a relation that is symmetric (F) and anti-symmetric (F).
   (A) $R = \{(a,b) \mid a = b\}$ (B) $R = \{(a,b) \mid a \bmod b = 0\}$
   (C) $R = \{(a,b) \mid a \bmod 2 = b \bmod 2\}$ (D) $R = \{(a,b) \mid a \bmod b \neq 0\}$

5. Convert the Hasse diagram  to matrix.

   (A) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

6. Convert the incidence matrix $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ to adjacency matrix.
   (A) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

7. Find the graph that has degree equals 90.

(A) K_{10} (B) P_{10} (C) C_{10} (D) $K_{10,10}$

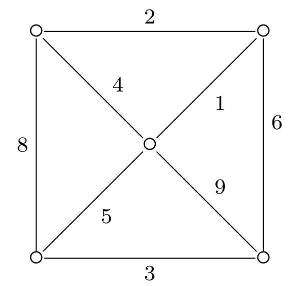
8. Find the distance matrix of P_4.

(A) $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 2 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$

9. Find the graph that is an Euler circuit.

(A) K_6 (B) $K_{6,4}$ (C) $K_{3,3}$ (D) $K_{3,4}$

10. Find the total weight of the Minimal Spanning Tree (MST) for this graph.



(A) 9 (B) 10 (C) 11 (D) 12

Part II. (5 points each) Write complete solutions.

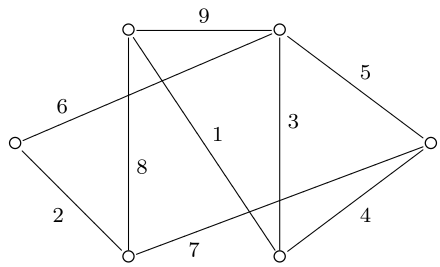
11. Convert the proposition $P \wedge (Q \to R)$ to DNF.

12. Count how many multiples of 12 or 15 from the number 1 to 200.

13. Given the matrix for the relation R, find the matrix for the transitive closure $\overline{R}$.

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

14. Solve the Chinese Postman Problem (CPP) for the given graph.



–Amin Witno