Department of Basic Sciences — Philadelphia University

Final Exam **Discrete Structures** **02–02–2017**

Part I. (2 points each) Circle one answer from the multiple choice.

1. Convert the DNF $(p \wedge q) \vee (\neg p \wedge q)$ to CNF.

   (A) $(\neg p \vee q) \wedge (p \vee q)$ (B) $(\neg p \vee q) \wedge (p \vee \neg q)$
   (C) $(\neg p \vee \neg q) \wedge (p \vee q)$ (D) $(\neg p \vee \neg q) \wedge (\neg p \vee q)$

2. Evaluate $\{1, 2, 4, 6\} \oplus \{2, 3, 4\}$.

   (A) $\{1, 2, 6\}$ (B) $\{1, 3, 6\}$ (C) $\{1, 5, 6\}$ (D) $\{1, 4, 5\}$

3. Compute $\gcd(221, 143)$.

   (A) 11 (B) 13 (C) 17 (D) 19

4. Which recurrence relation gives the sequence 0, 2, 4, 14, 40, ...?

   (A) $f(n) = 3f(n-1) + 2f(n-2)$ (B) $f(n) = 2f(n-1) + 3f(n-2)$
   (C) $f(n) = 3f(n-1) + 3f(n-2)$ (D) $f(n) = 2f(n-1) + 2f(n-2)$

5. Compute R^2 for the relation $R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

   (A) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. For $A = \{1, 2, 3, 4\}$, which relation is symmetric (F) and reflexive (T)?

   (A) $R = \{(x, y) \mid x \leq y\}$ (B) $R = \{(x, y) \mid x < y\}$
   (C) $R = \{(x, y) \mid x \bmod 2 = y \bmod 2\}$ (D) $R = \{(x, y) \mid x \neq y\}$

7. Convert the incidence matrix $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

   (A) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

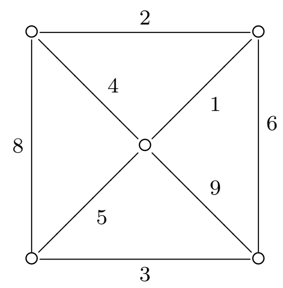
8. Which graph has degree 42?

   (A) $K_{6,4}$ (B) K_7 (C) K_8 (D) $K_{9,3}$

9. Which graph is an Euler circuit?

   (A) K_8 (B) K_7 (C) $K_{5,2}$ (D) $K_{4,3}$

10. Find the total weight of the Minimal Spanning Tree (MST) for this graph.



   (A) 9 (B) 10 (C) 11 (D) 12

Part II. (5 points each) Write complete solutions.

11. Count how many non-negative integer solutions for $A + B + C = 12$ such that $A \leq 3$ and $B \leq 4$.

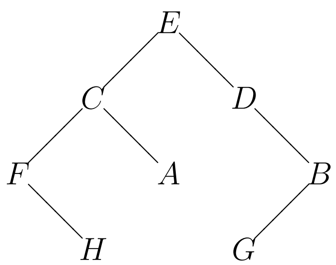
12. Let $A = \{2, 4, 6, 12, 18\}$ and $R = \{(x, y) \mid y \bmod x = 0\}$

    (a) Draw the graph of R.

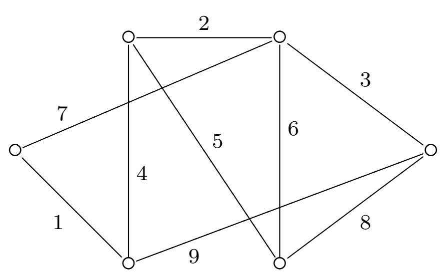
    (b) Prove that R is a partial order relation.

    (c) Draw the Hasse diagram.

13. Find the output using (a) pre-order (b) in-order (c) post-order algorithms for the given binary tree.



14. Solve the Chinese Postman Problem (CPP) for the given graph.



–Amin Witno