Department of Basic Sciences — Philadelphia University

Exam 2 **Discrete Structures** **27–12–2017**

Part I. (8 questions, 1 point each) Circle one answer.

1. Let $|A| = 19$. How many subsets of A have 3 elements?

   (A) 816 (B) 969 (C) 560 (D) 680

2. Count how many non-negative solutions in $A + B + C + D = 13$ with integers $A \geq 6$ and $B \geq 5$.

   (A) 10 (B) 20 (C) 35 (D) 56

3. Find the function $S(n)$ which gives the sequence 2, 3, 4, 2, 3, 4, 2, 3, 4, ...

   (A) $S(n) = 2 + (n \bmod 2)$ (B) $S(n) = 3 + (n \bmod 2)$

   (C) $S(n) = 2 + (n \bmod 3)$ (D) $S(n) = 3 + (n \bmod 3)$

4. Let $S(n) = 2\,S(n-1) + S(n-2)^2$ with $S(0) = 0$ and $S(1) = 1$. Find $S(4)$.

   (A) 14 (B) 23 (C) 32 (D) 40

5. Find the matrix corresponding to the relation $R = \{(x, y) \mid \lfloor \frac{x}{3} \rfloor = \lfloor \frac{y}{3} \rfloor\}$.

   (A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

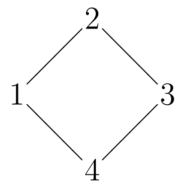
6. Convert the incidence matrix $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ to adjacency matrix.

   (A) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

7. If $R = \{(1,2),(2,4),(3,1),(4,2)\}$ and $S = \{(1,3),(2,1),(3,4),(4,2)\}$, find $S \circ R$.

(A) $\{(1,1),(2,2),(3,3),(4,1)\}$ (B) $\{(1,1),(2,2),(3,2),(4,4)\}$

(C) $\{(1,1),(2,2),(3,2),(4,1)\}$ (D) $\{(1,1),(2,2),(3,3),(4,4)\}$

8. Change the Hasse diagram 

to matrix.

(A)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(B)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(C)
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

(D)
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Part II. (3 questions, 4 points each) Write complete solution.

9. Find the formula for the function $S(n)$ given by the recurrence relation:

$$\begin{cases} S(n) = -2\,S(n-1) + 35\,S(n-2) \\ S(0) = 3 \\ S(1) = 7 \end{cases}$$

10. Use induction to prove the formula for all $n \geq 1$.

$$1 + 7 + 49 + \cdots + 7^n = \frac{7^{n+1} - 1}{6}$$

11. Given the relation matrix R, find the matrix for the transitive closure $\overline{R}$.

$$R = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

–Amin Witno