PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Module:	Modern Euclidean Geometry	Paper:	Exam 1
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Problems 1 to 7: Circle the best choice, 1 point each.

(1)
The negation of the statement S: “For every two points P and Q there is a unique line incident with P and Q” is the statement ~S: “For every two points P and Q there is more than one line incident with P and Q”. (a) true (b) false

(2)
In Elliptic geometry there are no parallel lines. (a) true (b) false

(3)
Three points A, B, C are collinear means that the lines AB and AC are parallel. (a) true (b) false

Problems 4 to 7: Consider the following model.
Points are A, B, C, D and Lines are {A, B}, {A, C}, {A, B, D}, {B, C, D}

(4)
In this model the Incidence Axiom 1 is (a) true (b) false.

(5)
In this model the Incidence Axiom 2 is (a) true (b) false.

(6)
In this model the Incidence Axiom 3 is (a) true (b) false.

(7)
This model satisfies the parallel postulate of (a) Euclidean (b) Elliptic (c) Hyperbolic geometry (d) none of them

Problems 8 to 13: Write the definitions, 1 point each.

(8)
The midpoint of two points A and B

(9)
the ray AB

(10)
opposite rays

(11)
the angle BAC

(12)

the interior of an angle

(13)

A and B are on the same side of a line l

(14)

Find a model with 3 points such that the Incidence Axioms 1, 2, 3 are all false. (1 point)

Points: A, B, C

Lines: ________________________________

(15)

Write the correct reasons to justify the steps of this proof, 0.5 point each.

Given A*B*C and B*C*D then A, B, C, D are all distinct and collinear and A*C*D.

Proof.

1. A, B, C are distinct and collinear (_________________)
2. B, C, D are distinct and collinear (same as 1)
3. Suppose A = D (proof by contradiction)
4. Then A*B*C = D*B*C
5. This is impossible because B*C*D (_________________)
6. So A ≠ D and A, B, C, D all distinct
7. Let A, B, C be on the line l and B, C, D be on the line l_2
8. Both l and l_2 pass through B and C, so l = l_2 (_________________)
9. So A, B, C, D on the line l, collinear
10. There exists a point P not on l (_________________)
11. There exists a line m passing through P and C (_________________)
12. Suppose A and B are on opposite sides of m (proof by contradiction)
13. Then segment AB intersects m (_________________)
14. This intersection must be C (_________________)
15. Then C belongs to segment AB, A*C*B
16. This is impossible because A*B*C (_________________)
17. So A and B are on the same side of m
18. B and D are on opposite sides of m (_________________)
19. So A and D are on opposite sides of m (_________________)
20. Then segment AD intersects m (same as 13)
21. This intersection must be C (same as 14)
22. So C belongs to segment AD, A*C*D

(16)

Prove the following proposition (3 points).

Given a line l and 3 distinct points A, B, C, not collinear. If l intersects the segment AB then l also intersects either segment AC or segment BC.