

**PHILADELPHIA UNIVERSITY**  
**DEPARTMENT OF BASIC SCIENCES**

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Module:	Modern Euclidean Geometry	Paper:	Final Exam
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Problems 1 to 6: Circle the best choice, 2 points each.

1. Three lines are concurrent; the meaning is
  - (a) they all intersect at one point
  - (b) they are all parallel
  - (c) they are all congruent to each other
  - (d) they are not parallel to each other
  
2. What is a definition of  $AB < CD$  ?
  - (a) there is E such that  $A*B*E$  and  $BE \approx CD$
  - (b) there is E such that  $C*D*E$  and  $CE \approx AB$
  - (c) there is E such that  $A*E*B$  and  $AE \approx CD$
  - (d) there is E such that  $C*E*D$  and  $CE \approx AB$
  
3. What is a bisector of  $\angle BAC$  ?
  - (a) a line that intersects both lines AB and AC
  - (b) a ray AD between AB and AC such that  $\angle BAD \approx \angle DAC$
  - (c) half of the angle BAC
  - (d) any point D which is interior of  $\angle BAC$
  
4. There exist 3 points which are not collinear. The negation of this statement is
  - (a) There are 3 points which are collinear.
  - (b) There are less than 3 points which are not collinear.
  - (c) Given 3 points, they are collinear.
  - (d) Given a line, there exist 3 points on the line.
  
5. Given a line l and a point P not on l. Which statement cannot be proved in Neutral Geometry?
  - (a) There exists a line through P perpendicular to l.
  - (b) There exists a unique line through P perpendicular to l.
  - (c) There exists at least one line through P parallel to l.
  - (d) There exists a unique line through P parallel to l.
  
6. AAA is a theorem in
  - (a) Neutral Geometry
  - (b) Hyperbolic Geometry only
  - (c) Euclidean Geometry only
  - (d) Euclidean and Hyperbolic Geometries

Problems 7 to 10 are related to the following model, 2 points each.

Points: A, B, C, D, E

Lines: {A, B}, {A, C}, {C, D}, {B, C, E}, {D, E}

7. In this model the Incidence Axiom 1 is (a) true (b) false.
8. In this model the Incidence Axiom 2 is (a) true (b) false.
9. In this model the Incidence Axiom 3 is (a) true (b) false.
10. This model satisfies the parallel postulate of
  - (a) Euclidean geometry
  - (b) Elliptic geometry
  - (c) Hyperbolic geometry
  - (d) none of them

Problems 11 to 13: Give the definitions, 2 points each.

11. the opposite of ray AB
12. A and B are on opposite sides of a line l
13.  $\angle BAC < \angle EDF$

Problems 14 to 16: Write the propositions in detail, 2 points each.

14. Angle Addition
15. SAA Criterion
16. Alternate Interior Angle Theorem

Problems 17 to 19 are related to the following axioms, 2 points each.

Axiom 1: There exist 2 parallel lines

Axiom 2: There exist 3 points which are collinear

Axiom 3: Given 2 points, there exists a unique line incident with them

17. Give a model such that Axioms 1, 2, 3 are all true.

18. Give a model such that only Axiom 1 is true, the others false.

19. Give a model such that only Axiom 2 is true, the others false.

20. Fill in the blanks to complete the proof, 2 points each.

Given  $\triangle ABC$  and  $\triangle ABD$  where C and D are on opposite sides of line AB.  
If  $AC \approx AD$  and  $BC \approx BD$  then  $\triangle ABC \approx \triangle ABD$ .

Proof.

1.  $\angle ACD \approx \angle ADC$  ( )
2.  $\angle BCD \approx \angle BDC$  ( same as 1 )
3.  $\angle ACB \approx \angle ADB$  ( )
4.  $\triangle ABC \approx \triangle ABD$  ( )

21. Prove the following proposition, 6 points each.

Given  $\angle BAC$  and a point D on the line BC.

Prove that if D is in the interior of  $\angle BAC$  then  $B^*C^*D$ .