

**Exam 1**

**Graph Theory**

**02-04-2017**

Part I. (1 point each) Write short answer.

1. The degree sequence of  $G$  is  $(4, 3, 2, 1, 1, 1)$ . Find the degree sequence of  $\overline{G}$ .

2. The degree sequence  $(5, 4, 4, 3, 3, 2, 1)$  is graphical. True or false?

(A) True

(B) False

3. Given the incidence matrix  $Z$  of a graph, find the adjacency matrix  $A$ .

$$Z = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

4. Which degree sequence comes from a tree? (Circle one answer.)

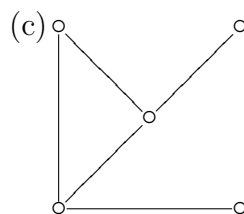
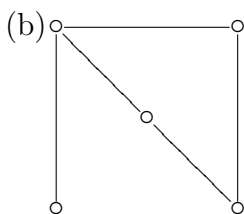
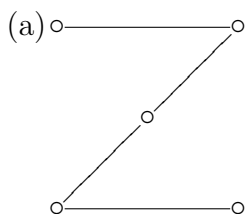
(A) 4,4,3,2,1,1,1

(B) 4,3,2,2,1,1,1

(C) 4,3,1,1,1,1,1

(D) none of these

5. Which graph is self-complementary? (Circle one answer.)



(d) none of these

6. Which graph contains  $C_5$ ? (Circle one answer.)

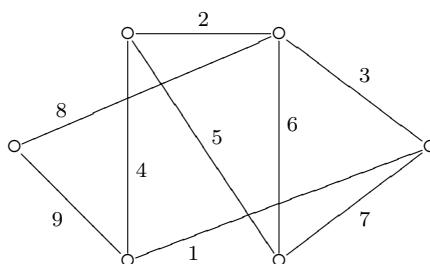
(A)  $C_6$

(B)  $K_{2,3}$

(C)  $P_6$

(D) none of these

7. Find the the minimal spanning tree (MST) and its value.



8. Which graph  $G$  is connected and  $\overline{G}$  is also connected? (Circle one answer.)

- (A)  $P_3$                       (B)  $K_5$                       (C)  $K_{3,3}$                       (D) none of these

9. A complete bipartite graph has degree 320. Find the minimum number of vertices.

10. A connected regular graph has 17 edges. Find the number of vertices.

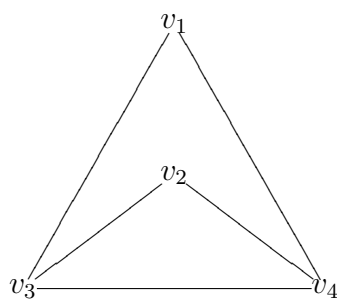
11. Which graph is cyclic and contains a bridge? (Circle one answer.)

- (A)  $K_{1,5}$                       (B)  $K_{3,3}$                       (C)  $C_6$                       (D) none of these

Part II. (3 points each) Write complete solution.

12. Draw 4 non-isomorphic trees with 6 vertices.

13. Find the number of spanning trees for the given labeled graph.



14. Let  $G$  be a tree. Prove that if  $G$  is self-complementary, then  $G \approx P_4$ .

–Amin Witno