Department of Basic Sciences — Philadelphia University

Exam 1 **Graph Theory** **24–03–2019**

Part I. Short Answer (1 point each)

1. If the graph G is 4-regular with 16 edges, then find $|V_G|$.

2. Prove the sequence $(5, 4, 4, 3, 3, 2, 2, 1)$ is graphical or not graphical.

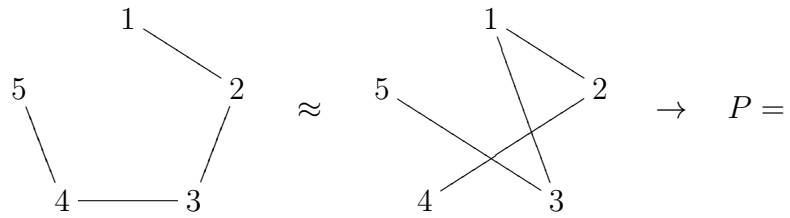
3. Find all values of n such that $C_n \subseteq \overline{K_{4,4}}$.

4. If the graph G is self-complementary with 16 vertices, then find $|E_G|$.

5. Given the incidence matrix Z of a graph, find the adjacency matrix A.

$$Z = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \rightarrow \quad A =$$

6. The two graphs below are isomorphic with adjacency matrices A and B. Find a permutation matrix P (without proof) such that $PAP^T = B$.



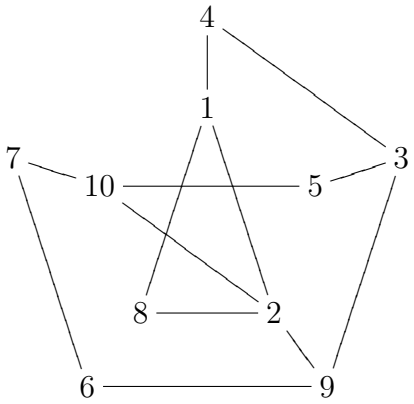
$\rightarrow \quad P =$

7. The degree sequence of G is $(9, 8, 8, 6, 5, 5, 4, 3, 3, 1)$. Determine the number of leaves in $\overline{G}$.

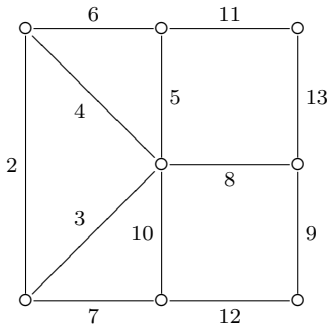
8. Draw two non-isomorphic trees with the same degree sequence $(4, 2, 2, 1, 1, 1, 1)$.

Part II. (4 points each) Write complete solution.

9. Draw the rooted and unrooted spanning trees of the labeled graph using (a) Breadth-First Search starting at vertex 1, and (b) rooted and unrooted spanning trees using Depth-First Search algorithms starting at vertex 1.

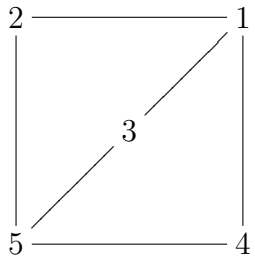


10. Apply (a) Prim's algorithm (v_1 is given) and (b) Kruskal's algorithm to produce a minimal spanning tree for the weighted graph and (c) compute its total value.



(a) v_1 (b)

11. Use the matrix tree algorithm to determine the number of labeled spanning trees of the graph.



–Amin Witno