Philadelphia University

Department of Basic Sciences

Exam 1 **Linear Algebra 2** **20–03–2019**

Choose 5 problems out of 6 and give complete solutions. No bonus.

1. Let $W = \{A \in M_{2\times 2} \mid \det A = 0\}$. Prove W is or is not a subspace of $M_{2\times 2}$.

2. Prove that the Wronskian of $\{e^x, e^{2x}, e^{3x}\}$ is $W(x) = 2e^{6x}$, and then explain why this set is linearly dependent or why independent in $F(-\infty, \infty)$.

3. Find all the values of $k \in \mathbb{R}$ such that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ span $\mathbb{R}^3$, where

$$\begin{aligned}\mathbf{u} &= (\tfrac{1}{2}, -\tfrac{1}{2}, 1)\\ \mathbf{v} &= (1, k, 1)\\ \mathbf{w} &= (k, 1, 3)\end{aligned}$$

4. Prove that $\{p_1, p_2, p_3, p_4\}$ is linearly dependent in P_3 by writing one of these polynomials as a linear combination of the others, where

$$\begin{aligned}p_1 &= 1 - x + x^2 + x^3\\ p_2 &= 1 - x^3\\ p_3 &= -2x + 4x^2 + 3x^3\\ p_4 &= x + x^2 - 3x^3\end{aligned}$$

5. Find the dimension and a basis for the given vector space.

   (a) The plane in $\mathbb{R}^3$ given by the equation $2x + 3y = 0$.

   (b) The subspace of all 2×2 symmetric matrices with real entries.

   (c) The solution space of the linear system $A\mathbf{x} = \mathbf{0}$, where

$$A = \begin{bmatrix} 1 & 1 & 2 & -3 & 5\\ 0 & 1 & 1 & -1 & 0\\ 0 & 0 & 0 & 3 & 3 \end{bmatrix}$$

6. Let $\{v_1, v_2, v_3\}$ be a basis for a vector space V, and let

$$\begin{aligned}w_1 &= v_1 - v_3\\ w_2 &= 3v_1 + 2v_2\\ w_3 &= v_1 + v_2 + v_3\end{aligned}$$

   Prove that $\{w_1, w_2, w_3\}$ is also a basis for V.

–Amin Witno