Philadelphia University

Department of Basic Sciences

Exam 1 **Number Theory** **3–4–2007**

Each problem is worth 2 points. Solutions must be complete to receive full credit.

1. Illustrate Fermat Factorization with $n = 3569$.
2. Does the equation $36x + 114y = 82$ have a solution? Why or why not?
3. I made two calls today using my Fastlink account, one call to another Fastlink customer for 7 piasters per minute and another call to a MobileCom number for 12 piasters per minute. The total charge was one dinar and 37 piasters. For how long did I talk in each call? Use linear equation theorem to solve this problem.
4. Count how many positive divisors of the number 2,000,000.
5. Are there infinitely many primes in the sequence $46, 49, 52, 55, 58, 61, 64 \ldots$? Why or why not?
6. Find two Sophie Germain primes between 50 and 100.
7. Estimate how many prime numbers below 100,000.
8. *Proposition:* If p is a prime and $p \mid n^2$ then $p^2 \mid n^2$. Give an example where this proposition is false when p is not a prime.
9. *Euclid's Lemma* says that if $d \mid mn$ and d is relatively prime to n then $d \mid m$. Prove it.
10. Prove that there are no prime triplets except 3, 5, 7. Hint: Use residues mod 6.

The list of primes below 200.

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				

–Amin Witno