Philadelphia University

Department of Basic Sciences

Exam 2 **Number Theory** **12–12–2007**

Each problem is worth 4 points. Solutions must be complete to receive full credit.

1. Illustrate the Successive Squaring algorithm to compute

$$2^{146} \quad \% \quad 49$$

2. Compute with the help of Wilson's theorem:

$$38! \quad \% \quad 41$$

Note that 41 is prime.

3. Solve the following system of three congruences:

$$\begin{aligned} x &\equiv 3 \pmod 4 \\ x &\equiv 5 \pmod 9 \\ x &\equiv 7 \pmod{11} \end{aligned}$$

4. Find all the solutions for x such that

$$x^{29} \equiv 88 \pmod{91}$$

Note that $91 = 7 \times 13$.

5. Prove that if $\gcd(a, 55) = 1$ then

$$a^{20} \equiv 1 \pmod{55}$$

Hint: you will need the Chinese remainder theorem and Fermat's little theorem.

–Amin Witno