Philadelphia University

Department of Basic Sciences

Final Exam **Number Theory** **03–02–2008**

1. Evaluate $56! \,\%\, 59$. The number 59 is prime.

2. Solve the following system of three congruences:

   $x \equiv 3 \pmod 7$
   $x \equiv 5 \pmod 8$
   $x \equiv 7 \pmod{15}$

3. Find all the solutions to $x^{29} \equiv 52 \pmod{95}$. Note that $95 = 5 \times 19$.

4. Is 13 a primitive root modulo 257? Why or why not? The number 257 is prime.

5. Complete the following table and use it to solve $2^x \equiv 9 \pmod{17}$.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$5^k \,\%\, 17$																

6. Evaluate the Legendre symbol $\left(\frac{296}{313}\right)$.

7. Prove that $a^{31} \equiv a \pmod{77}$ for any integer a.

8. Prove that $\phi(n)$ is even for all $n > 2$.

–Amin Witno