

PHILADELPHIA UNIVERSITY  
DEPARTMENT OF BASIC SCIENCES

Final Exam

Number Theory

08–06–2014

Write complete solution for each problem.

1. Solve the linear congruence  $123x \equiv 9 \pmod{357}$ .
2. Solve the system of three congruences.

$$\begin{cases} x \equiv 1 \pmod{7} \\ x \equiv 5 \pmod{9} \\ x \equiv 9 \pmod{10} \end{cases}$$

3. Solve the congruence  $x^{43} \equiv 5 \pmod{77}$ .
4. Solve the congruence  $3^x \equiv 5 \pmod{11}$ .
5. Solve the quadratic congruence  $x^2 \equiv 14 \pmod{55}$ .
6. Is the number 186 a quadratic residue (QR) or nonresidue (NR) modulo 239? The number 239 is prime.
7. Let  $\gcd(a, 55) = 1$ . Prove that  $a^{20} \equiv 1 \pmod{55}$ . Hint: use the Chinese remainder theorem (CRT) and Fermat's little theorem (FLT).

-Amin Witno

The list of prime numbers  $p < 200$ :

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				