

PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Final Exam

Number Theory

04–02–2015

Solutions must be complete in order to receive full credit.

1. Let $\gcd(m, n) = 1$. Prove that if $m \mid x$ and $n \mid x$, then $mn \mid x$.
2. Compute $195! \% 199$ using Wilson's theorem.
3. Find all the integer solutions for the system of three congruences.

$$\begin{aligned}x &\equiv 3 \pmod{5} \\x &\equiv 2 \pmod{6} \\x &\equiv 1 \pmod{7}\end{aligned}$$

4. Compute $2^{894} \% 35$ using Euler's theorem.
5. Find all the integer solutions for the congruence $x^{13} \equiv 5 \pmod{32}$.
6. Find all the integer solutions for the congruence $10^x \equiv 3 \pmod{13}$ using the primitive root $g = 2$.
7. Evaluate the Legendre symbol $\left(\frac{416}{433}\right)$.
8. Find all the integer solutions for the congruence $x^2 \equiv 31 \pmod{55}$.

—Amin Witno

The list of primes below 200.

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199				