

Department of Basic Sciences — Philadelphia University

Mid Exam

Number Theory

12–12–2021

1. (3 points) Prove the theorem: Let $\gcd(a, b) = 1$. If $a \mid bc$ then $a \mid c$
2. (2 points) Prove 211 is prime or composite using Trial Division.
3. (2 points) Count how many divisors of the number 5040
4. (3 points) Use Fermat factorization algorithm to factor $n = 5917$
5. (4 points) Find the congruence class solution of $81x \equiv 27 \pmod{144}$
6. (4 points) Solve the system of linear congruences
$$\begin{cases} x \equiv 5 \pmod{11} \\ x \equiv 7 \pmod{8} \end{cases}$$
7. (4 points) Compute $79! \% 83$ using Wilson's theorem.
8. (4 points) Use SSA to compute $9^{101} \% 100$
9. (4 points) Prove that $221 \mid n^{49} - n$ for all $n \in \mathbb{Z}$ (Hint: $221 = 13 \times 17$)

–Amin Witno