

Midterm Exam

Number Theory

11/12/2022

1. (3 points) Prove the theorem: Let $\gcd(a, b) = 1$. If $a \mid c$ and $b \mid c$ then $ab \mid c$
2. (2 points) Prove 241 is prime or composite using Trial Division.
3. (2 points) Count how many divisors of the number 2268
4. (3 points) Use Fermat factorization algorithm to factor $n = 6161$
5. (4 points) Find the congruence class solution of $63x \equiv 18 \pmod{144}$
6. (4 points) Solve the system of linear congruences
$$\begin{cases} x \equiv 5 \pmod{25} \\ x \equiv 7 \pmod{9} \end{cases}$$
7. (4 points) Compute $75! \% 79$ using Wilson's theorem.
8. (4 points) Use SSA to compute $7^{98} \% 100$
9. (4 points) Prove that $95 \mid n^{37} - n$ for all $n \in \mathbb{Z}$

–Amin Witno