Philadelphia University

Department of Basic Sciences

Exam 1 **Numerical Analysis** **16–11–2006**

1. (a) Find the third Taylor polynomial for $f(x) = e^{-x}$ about the center $x = 0$.
   (b) Use it to approximate the value of $e^{-0.5}$.
   (c) What is the error bound in this approximation?
2. (a) Use Bisection Method to approximate the value of $\sqrt[3]{2}$ on the interval $(1, 2)$ until you find p_4.
   (b) How many iterations are needed to have error less than 10^{-7}?
3. Let $f(x) = x^2 - 5$ and $g(x) = x - \frac{f(x)}{f'(x)}$ with given interval $(2, 3)$.
   (a) Show that p is a root of f if and only if p is a fixed point of g.
   (b) Use Fixed Point iterations for $g(x)$ with $p_0 = 2.5$ until you find p_3
   (c) Show that $g(x)$ satisfies the conditions of Fixed Point Theorem.
   (d) How many iterations are needed to have error less than 10^{-5}?
4. Let $f(x) = e^{2x} - 2x^2 - 2x - 1$ with given interval $(-1, 1)$.
   (a) Use the Secant Method to find a root of f with $p_0 = -1$ and $p_1 = 1$ until you find p_3.
   (b) Use Newton's Method with $p_0 = -1$ until you find p_3.
   (c) Show that $p = 0$ is a root of $f(x)$ and find its multiplicity m.
   (d) Since $m > 1$ we should replace $f(x)$ by $\mu(x) = \frac{f(x)}{f'(x)}$ before applying Newton's Method. What will be the new iteration function $g(x)$? (You are not asked to do the iteration, only find g.)

–Amin Witno