# Philadelphia University

## Department of Basic Sciences

**Midterm Exam** **Numerical Analysis** **16–05–2022**

1. (a) (3 points) Given $f(x) \in C[3.2, 4]$, find $p_2$ using Bisection method.

$$f(x) = x^3 - 7x^2 + 14x - 6$$

   (b) (3 points) What is the minimum iterations $n$ for the approximation $p_n$ to be accurate within $10^{-10}$?

2. (a) (2 points) Show that $f(x) = 0$ if and only if $g(x) = x$.

$$f(x) = x^5 - x^2 - 3; \quad g(x) = \sqrt{\frac{2x^2 + 3}{x^3 + 1}}$$

   (b) (2 points) Given $p_0 = 1.5$, find $p_2$ using the Fixed-Point Iteration method.

3. (a) (2 points) Given $f(x) = \sin x - e^{-x}$ and $p_0 = 1$, find $p_1$ using Newton method.

   (b) (2 points) Using $p_0 = 1$ and $p_1 = 0.5$, find $p_2$ using Secant method.

4. (6 points) Use Horner method to find $p_1$ as a rational number, with $p_0 = \frac{1}{2}$

$$P(x) = x^4 - 2x^3 + 1$$

5. (4 points) Use Neville method to approximate $f(1.5)$ using Lagrange polynomial degree 2.

| $n$ | $x$ | $f(x)$ | deg 1 | deg 2 |
|---|---|---|---|---|
| 0 | 1.0 | 2.7536 | | |
| 1 | 1.3 | 3.2740 | $(0,1) = 3.6209$ | |
| 2 | 1.6 | 3.7985 | $(1,2) = ?$ | $(0,1,2) = ?$ |

6. (6 points) Given $f(x) = x + \frac{1}{x}$ with $x_0 = \frac{1}{3}$, $x_1 = 1$, $x_2 = 2$, find the Lagrange polynomial of degree 2. (Final answer in the form $Ax^2 + Bx + C$ where $A, B, C$ are rational numbers.)

–Amin Witno