Philadelphia University
Department of Basic Sciences

Final Exam **Problem Solving** **05–02–2015**

1. Find the following sum with proof. Express your answer as a single fraction $\frac{f(n)}{g(n)}$.
$$\frac{1}{3\cdot 4\cdot 5}+\frac{2}{4\cdot 5\cdot 6}+\frac{3}{5\cdot 6\cdot 7}+\cdots+\frac{n-3}{(n-1)n(n+1)}$$

2. Given that
$$1+2+3+\cdots+n=\frac{n(n+1)}{2}$$
$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$
Derive the formula for the sum $1^3+2^3+3^3+\cdots+n^3$.

3. Write the identity for the following pattern (without proof).
$$1+2=3$$
$$4+5+6=7+8$$
$$9+10+11+12=13+14+15$$
$$16+17+18+19+20=21+22+23+24$$
$$\cdots=\cdots$$

4. Prove the following identity involving the binomial coefficients.
$$\binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k}$$

5. Find the formula for S_n given that $S_0=2$ and $S_1=1$, and the recurrence relation $S_n=S_{n-1}+S_{n-2}$ for $n\geq 2$.

6. Let the Fibonacci numbers F_n be defined by $F_0=F_1=1$ and $F_n=F_{n-1}+F_{n-2}$ for $n\geq 2$. Prove the identity $\sum_{k=0}^{n} F_k^2=F_nF_{n+1}$.

7. Find the identity for the finite sum $F_1+F_3+F_5+F_7+\cdots$ and prove it.

8. Write the identity for the following pattern involving F_n and prove it.
$$1-1+2=1+1$$
$$1-1+2-3=1-2$$
$$1-1+2-3+5=1+3$$
$$1-1+2-3+5-8=1-5$$
$$1-1+2-3+5-8+13=1+8$$
$$\cdots=\cdots$$

–Amin Witno