## Midterm Exam Probability Theory 19/12/2022

All answers must be in reduced fractions, or rounded in decimal to 2 significant digits.

- 1. (1pt) In the next 2026 World Cup, there will be 48 football teams competing for first, second, and third place. Count the number of possible outcomes.
- 2. (1pt) In a supermarket there are 59 apples, and 8 of them are bad. If we randomly pick 2 apples, compute the probability that both are bad.
- 3. (5pt) A coin is tossed 6 times. Let  $A = \{ \text{at least 5 Heads} \}$  and  $B = \{ \text{the first is Heads} \}$ .
  - (a) Compute P(A)
  - (b) Compute P(B)
  - (c) Compute  $P(A \cap B)$
  - (d) Compute  $P(A \cup B)$
  - (e) Prove A and B are dependent or independent.
- 4. (2pt) An airline flight has a probability 34% of getting delayed when it is raining, and 19% when not raining. The probability that today will rain is 81%. Find the probability that this flight will be delayed today.
- 5. (2pt) Assume that 68% of the Samsung mobile phones in the country come from China, 9% from Malaysia, and 23% from India. From China, an average 3.2% of the phones are defective, from Malaysia 2.9%, and from India 1.7%. Given that a phone is found defective, what is the probability it came from India?
- 6. (2pt) Given the probability density function f(x). Compute P(X > 1).

$$f(x) = \begin{cases} \frac{1}{4}x^3 & \text{for } 0 < x < 2\\ 0 & \text{else} \end{cases}$$

7. (2pt) Given the probability density function f(x). Find F(x).

$$f(x) = \begin{cases} \frac{1}{6\sqrt{x}} & \text{for } 0 < x < 9\\ 0 & \text{else} \end{cases}$$

8. (2pt) Given the distribution function F(x) for a discrete random variable X.

$$F(x) = \begin{cases} 0 & \text{for } x < 1\\ \frac{1}{4} & \text{for } 1 \le x < 3\\ \frac{3}{5} & \text{for } 3 \le x < 5\\ 1 & \text{for } x \ge 5 \end{cases}$$

(a) Find  $P(X \le 3)$ 

(b) Find P(X = 3)

- 9. (2pt) Given the joint probability distribution  $f(x, y) = k \begin{pmatrix} x \\ y \end{pmatrix}$  where  $x \in \{3, 4\}$  and  $y \in \{1, 2\}$ .
  - (a) Find k
  - (b) Find F(3,3)
- 10. (3pt) Given the joint probability density function f(x, y). Compute  $P(X, Y \leq \frac{3}{2})$ .

$$f(x,y) = \begin{cases} \frac{1}{3} \left( x + y \right) & \text{for } 0 < x < 1; 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

11. (4pt) Given the joint probability density function f(x, y). Find k.

$$f(x,y) = \begin{cases} k(x^2 - xy) & \text{for } x < 1; -x < y < x \\ 0 & \text{otherwise} \end{cases}$$

12. (4pt) Given the joint distribution function F(x, y). Compute P(X + Y > 3).

$$F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & \text{for } x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$