Final Exam **Probability Theory** **21/06/2023**

Each problem is worth 4 points.

1. Given the joint probability density function $f(x,y)$. Find $P(X < \frac{1}{2};\ Y < \frac{3}{2})$.

$$f(x,y) = \begin{cases} \frac{4}{3}xy & \text{for } 0 < x < 1;\ 1 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

2. Given the joint distribution function $F(x,y)$. Find $P(1 < X < 2;\ Y \leq 2)$.

$$F(x,y) = \begin{cases} 1 - e^{-x} - e^{-y} + e^{-x-y} & \text{for } x, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

3. Given the joint probability density function $f(x,y)$. Find the marginal density of X.

$$f(x,y) = \begin{cases} \frac{3}{2}xy & \text{for } x, y > 0;\ x + y < 2 \\ 0 & \text{otherwise} \end{cases}$$

4. Given the joint probability density function $f(x,y)$. Find the conditional density of X given $(Y = y)$, then compute $P(X < 1 \mid Y = \frac{1}{2})$.

$$f(x,y) = \begin{cases} \frac{1}{5}(2x + y) & \text{for } 0 < x < 2;\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

5. Given the discrete uniform distribution $f(x) = \frac{1}{3}$ with domain $x \in \{-1, 1, 2\}$. Compute μ and σ^2

6. Given the Pareto distribution $f(x) = \frac{2}{x^3}$ with domain $x \in (1, \infty)$. Compute μ and σ^2.

7. Given that $\sigma_X^2 = 3$, $\sigma_Y^2 = 4$, $\sigma_Z^2 = 1$ and $\sigma_{XY} = 3$, $\sigma_{XZ} = -2$, $\sigma_{YZ} = 1$. Let $W = X + 2Y - 3Z$. Compute σ_W^2.

8. Given the joint probability distribution $f(x,y)$. Compute σ_{XY}.

(x,y)	(−1,1)	(0,1)	(1,0)	(1,1)
$f(x,y)$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{2}$

9. Given the joint probability density function $f(x,y)$. Compute σ_{XY}.

$$f(x,y) = \begin{cases} x + y & \text{for } 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}$$

10. About 6.2% of the computers in the University are still using Windows 7. Estimate the probability that 5 out of 500 computers are using Windows 7, using the Poisson distribution $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ where $\lambda = pn$.

–Amin Witno