PHILADELPHIA UNIVERSITY
DEPARTMENT OF BASIC SCIENCES

Exam 2 **Set Theory** **11–12–2012**

To receive full credit, each solution must be correct and complete.

1. Answer True or False.
   (a) $\exists x \in \mathbb{R} : x^2 < x$
   (b) $\forall x \in \mathbb{R} : (x-2)^2 + 1 > 0$
   (c) $\exists! x \in \mathbb{Z} : (x-2)^2 = 25$
   (d) $\exists$ prime $p : p \bmod 4 = 0$
2. Use contradiction to prove that $\sqrt{2}$ is irrational.
3. Use induction to prove the formula for all $n \in \mathbb{N}$.
$$1 + 4 + 7 + 10 + \cdots + (3n-2) = \frac{3n^2 - n}{2}$$
4. Let $A = \{1, 3, 5, 7, 9, 11\}$ and $R = \{(a, b) \mid a \bmod 3 = b \bmod 3\}$. Prove that R is an equivalence relation on A and find all the equivalence classes.
5. Let $A = \{2, 3, 4, 6, 8, 12\}$ and $R = \{(a, b) \mid \frac{b}{a} \in \mathbb{Z}\}$. Prove that R is partial order relation on A and draw the Hasse diagram.

–Amin Witno