Homework 1 — Linear Algebra

1. Find all the solutions of the linear system using augmented matrix.

(a) $\begin{cases} 3x + 2y = 0 \\ x - y = 5 \end{cases}$ (b) $\begin{cases} 6x + 4y = -4 \\ -x - 2y = 0 \end{cases}$ (c) $\begin{cases} x - 2y = 0 \\ x - 4y = 8 \end{cases}$

2. Solve the system given by the augmented matrix, already in row-echelon form.

(a) $A = \begin{bmatrix} 1 & 2 & -4 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 & 4 & 7 & 10 \\ 0 & 1 & -3 & -4 & -2 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 5 & -4 & 0 & -7 & -5 \\ 0 & 0 & 1 & 1 & 7 & 3 \\ 0 & 0 & 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3. Solve the system of linear equations using Gaussian algorithm.

(a) $\begin{cases} x + y + 2z = 8 \\ -x - 2y + 3z = 1 \\ 3x - 7y + 4z = 10 \end{cases}$ (b) $\begin{cases} x + 2y + 5z = -1 \\ 2x + 4y - 8z = 4 \\ 3x - 9z = 5 \end{cases}$

(c) $\begin{cases} x + y + 2z + w = -1 \\ y - 3z + 2w = 2 \\ x + y - w = 3 \\ 5x + 3y + 2z + 2w = 4 \end{cases}$ (d) $\begin{cases} 3x + y + z + w = 0 \\ 5x - y + z - w = 0 \\ -x + y + w = 0 \end{cases}$

4. Repeat Problem (3) using Gauss-Jordan algorithm.

5. Find the values of k such that the system has (a) no solution (b) unique solution (c) many solutions.

$$\begin{cases} x + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y + (k^2 - 14)z = k + 2 \end{cases}$$

Homework 2 — Linear Algebra

1. Find a, b, c, d such that $\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$. Compute

(a) the first row of AB (b) the third row of AB (c) the second column of AB (d) the first column of BA (e) the third row of AA (f) the third column of AA

3. Compute (a) AB (b) $D+E$ (c) $D-E$ (d) DE (e) ED (f) $CA+B$.

$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$ $E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

4. Continue Problem (3) and compute, when possible,
   (a) $3C - D$ (b) $3ED$ (c) $(AB)C$
   (d) $A(BC)$ (e) $4BC + 2B$ (f) $D + E^2$

5. Continue Problem (3) and compute, when possible,
   (a) $2A^T + C$ (b) $D^T - E^T$ (c) $(D - E)^T$
   (d) $B - B^T$ (e) CC^T (f) $(DA)^T$

6. Continue Problem (3) and compute, if possible, the trace of
   (a) D (b) AC (c) CA
   (d) BC (e) $D - 3E$ (f) DE^T

Homework 3 — Linear Algebra

1. Compute A^{-1} using the 2x2 formula.
   (a) $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$ (c) $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (d) $A = \begin{bmatrix} 6 & 4 \\ -2 & -1 \end{bmatrix}$

2. Solve the system of linear equations using matrix inverse.
   (a) $\begin{cases} 3x + y = -1 \\ 5x + 2y = 3 \end{cases}$ (b) $\begin{cases} 3x + y = 5 \\ 5x + 2y = 7 \end{cases}$

   (c) $\begin{cases} -x + 5y = 2 \\ -x - 3y = -2 \end{cases}$ (d) $\begin{cases} 6x + 4y = 0 \\ -2x - y = -2 \end{cases}$

3. Repeat Problem (1) using row operations.

4. Compute A^{-1} using row operations, if exists.
   (a) $\begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 4 & 0 \\ 1 & 2 & 4 & 8 \end{bmatrix}$ (f) $\begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & -5 \end{bmatrix}$ (g) $\begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 3 & 0 \\ 2 & 1 & 5 & -3 \end{bmatrix}$

5. Solve the system of linear equations using matrix inverse.
   (a) $\begin{cases} x + 2y + 2z = -1 \\ x + 3y + z = 4 \\ x + 3y + 2z = 3 \end{cases}$ (b) $\begin{cases} 2x + y + z = 7 \\ 3x + 2y + z = -3 \\ y + z = 5 \end{cases}$

   (c) $\begin{cases} 2x + y + z = 2 \\ 3x + 2y + z = 1 \\ y + z = 3 \end{cases}$ (d) $\begin{cases} 3x + y + 7z + 9w = 4 \\ x + y + 4z + 4w = 7 \\ -x - 2z - 3w = 0 \\ -2x - y - 4z - 6w = 6 \end{cases}$

6. Express each matrix as the product of elementary matrices.
   (a) $\begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Homework 4 — Linear Algebra

1. Compute the determinant using the 2x2 formula.
(a) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 7 \\ -8 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{bmatrix}$

2. Compute the determinant using the 3x3 formula.
(a) $\begin{bmatrix} 1 & -2 & 7 \\ 3 & 5 & 1 \\ 4 & 3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 2 & -1 \\ -3 & 4 & -6 \\ 1 & 7 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 3 \\ 4 & 0 & -1 \\ 2 & 8 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{bmatrix}$

3. Compute the determinant in Problem (2c) using cofactor expansion along
(a) the first row (b) the second row (c) the second column (d) the third column

4. Compute the determinant in Problem (2d) using cofactor expansion along
(a) the first row (b) the second row (c) the second column (d) the third column

5. Compute the determinant using cofactor expansion.
(a) $\begin{bmatrix} 1 & 4 & -3 & 1 \\ 2 & 0 & 6 & 3 \\ 4 & -1 & 2 & 5 \\ 1 & 0 & -2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

Homework 5 — Linear Algebra

1. Compute the determinant using row operations.
(a) $\begin{bmatrix} 2 & 3 & 7 \\ 0 & 0 & -3 \\ 1 & -2 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & 1 \\ 4 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -2 & 0 \\ -3 & 5 & 1 \\ 4 & -3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -4 & 8 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

2. Compute the determinant using row operations.
(a) $\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 6 & 9 & 3 \\ -1 & 0 & 1 & 0 \\ 1 & 3 & 2 & -1 \\ -1 & -2 & -2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

3. Compute the determinant given that $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 5$.
(a) $\begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}$ (b) $\begin{bmatrix} -a & -b & -c \\ 2d & 2e & 2f \\ -g & -h & -i \end{bmatrix}$
(c) $\begin{bmatrix} g & h & i \\ d & e & f \\ a+d & b+e & c+f \end{bmatrix}$ (d) $\begin{bmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ 2g & 2h & 2i \end{bmatrix}$

4. Solve the system of linear equations using Cramer's rule.

(a) $\begin{cases} 4x + 5y = 2 \\ 11x + y + 2z = 3 \\ x + 5y + 2z = 1 \end{cases}$ (b) $\begin{cases} x + y - 2z = 1 \\ 2x - y + z = 2 \\ x - 2y - 4z = -4 \end{cases}$

(c) $\begin{cases} 3x - 4y = -5 \\ 2x + y = 4 \end{cases}$ (d) $\begin{cases} 2x - y + z - 4w = -32 \\ 7x + 2y + 9z - w = 14 \\ 3x - y + z + w = 11 \\ x + y - 4z - 2w = -4 \end{cases}$

Homework 6 — Linear Algebra

1. Determine invertible or not invertible using determinant.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 6 & 7 \\ 0 & 8 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 1 & -4 \\ 1 & 1 & 2 \\ 3 & 1 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 2 & 1 \\ 7 & 2 & 1 \\ 3 & 6 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 7 & 5 \\ 0 & 1 & -1 \\ 0 & 3 & 2 \end{bmatrix}$

2. Find k such that the matrix A is invertible.

(a) $\begin{bmatrix} k-3 & -2 \\ -2 & k-2 \end{bmatrix}$ (b) $\begin{bmatrix} k & 2 \\ 2 & k \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ k & 3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 0 \\ k & 1 & k \\ 0 & 2 & 1 \end{bmatrix}$

3. Find the inverse, if exists, using the adjoint method.

(a) $\begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 & 0 \\ 8 & 1 & 0 \\ -5 & 3 & 6 \end{bmatrix}$

4. Compute the determinant given that $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 5$.

(a) $\det(3A)$ (b) $\det(A^{-1})$ (c) $\det(2A^{-1})$ (d) $\det((2A)^{-1})$ (e) $\det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$

Homework 7 — Linear Algebra

1. Let $\mathbf{u} = (4, -1)$ and $\mathbf{v} = (0, 5)$ and $\mathbf{w} = (-3, -3)$. Evaluate
(a) $\mathbf{u} + \mathbf{w}$ (b) $\mathbf{v} - 3\mathbf{u}$ (c) $2(\mathbf{u} - 5\mathbf{w})$ (d) $3\mathbf{v} - 2(\mathbf{u} + 2\mathbf{w})$

2. Let $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (2, -3, 1)$ and $\mathbf{w} = (3, 2, -1)$. Evaluate
(a) $\mathbf{u} - \mathbf{w}$ (b) $-3\mathbf{v} - 8\mathbf{w}$ (c) $3(\mathbf{u} - 7\mathbf{v})$ (d) $2\mathbf{v} - (\mathbf{u} + \mathbf{w})$

3. Let $\mathbf{u} = (2, -2, 3)$ and $\mathbf{v} = (1 - 3, 4)$ and $\mathbf{w} = (3, 6, -4)$. Evaluate
(a) $\|\mathbf{u} + \mathbf{v}\|$ (b) $\|\mathbf{u}\| + \|\mathbf{v}\|$ (c) $\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$ (d) $\|(\|\mathbf{u} - \mathbf{v}\|\mathbf{w})\|$

4. Compute $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u} \cdot \mathbf{u}$ and $\mathbf{v} \cdot \mathbf{v}$.
(a) $\mathbf{u} = (1, 2),\ \mathbf{v} = (6, -8)$
(b) $\mathbf{u} = (7, 3, 5),\ \mathbf{v} = (-8, 4, 2)$
(c) $\mathbf{u} = (1, 1, 4, 6),\ \mathbf{v} = (2, -2, 3, -2)$
(d) $\mathbf{u} = (1, 1, -2, 3),\ \mathbf{v} = (-1, 0, 5, 1)$

5. Compute $d(\mathbf{u}, \mathbf{v})$ and the angle between the two vectors.
   (a) $\mathbf{u} = (1, 2),\ \mathbf{v} = (6, -8)$
   (b) $\mathbf{u} = (-3, 1, 2),\ \mathbf{v} = (4, 2, 5)$
   (c) $\mathbf{u} = (0, -2, -1, 2),\ \mathbf{v} = (-3, 2, 4, 4)$
   (d) $\mathbf{u} = (1, 2, -3, 0),\ \mathbf{v} = (5, 1, 2, -2)$

6. Let $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (2, -3, 1)$ and $\mathbf{w} = (3, 2, -1)$. Evaluate
   (a) $\mathbf{u} \cdot (7\mathbf{v} + \mathbf{w})$ (b) $\|(\mathbf{u} \cdot \mathbf{w})\mathbf{w}\|$ (c) $\|\mathbf{u}\|(\mathbf{v} \cdot \mathbf{w})$ (d) $(\|\mathbf{u}\|\mathbf{v}) \cdot \mathbf{w}$.

Homework 8 Linear Algebra

1. Determine orthogonal or not orthogonal for the two vectors.
   (a) $\mathbf{u} = (3, -2, 1, 3),\ \mathbf{v} = (-4, 1, -3, 8)$
   (b) $\mathbf{u} = (5, -4, 0, 3),\ \mathbf{v} = (-4, 1, -3, 8)$

2. Find a point-normal form of the equation of the plane passing through P with normal $\mathbf{n}$.
   (a) $P = (-1, 3, -2),\ \mathbf{n} = (-2, 1, -1)$
   (b) $P = (1, 1, 4),\ \mathbf{n} = (1, 9, 8)$
   (c) $P = (2, 0, 0),\ \mathbf{n} = (0, 0, 2)$
   (d) $P = (0, 0, 0),\ \mathbf{n} = (1, 2, 3)$

3. Determine parallel or not parallel for the two planes.
   (a) $4x - y + 2z = 5$ and $7x - 3y + 4z = 8$
   (b) $x - 4y - 3z - 2 = 0$ and $3x - 12y - 9z - 7 = 0$
   (c) $2y = 8x - 4z + 5$ and $x = \frac{1}{2}z + \frac{1}{4}y$
   (d) $(-4, 1, 2) \cdot (x, y, z) = 0$ and $(8, -2, -4) \cdot (x, y, z) = 0$

4. Determine perpendicular or not perpendicular for the two planes.
   (a) $3x - y + z = 4$ and $x + 2z = -1$
   (b) $x - 2y + 3z = 4$ and $-2x + 5y + 4z = -1$

5. Find $\|\text{proj}_{\mathbf{x}}\mathbf{u}\|$.
   (a) $\mathbf{u} = (1, -2),\ \mathbf{x} = (-4, -3)$
   (b) $\mathbf{u} = (3, 0, 4),\ \mathbf{x} = (2, 3, 3)$
   (c) $\mathbf{u} = (3, -2, 6),\ \mathbf{x} = (1, 2, -7)$
   (d) $\mathbf{u} = (2, 1, 1, 2),\ \mathbf{x} = (4, -4, 2, -2)$

6. Compute the distance between the point and the line.
   (a) $(-3, 1),\ 4x + 3y + 4 = 0$
   (b) $(-1, 4),\ x - 3y + 2 = 0$
   (c) $(2, -5),\ y = -4x + 2$
   (d) $(1, 8),\ 3x + y = 5$

7. Compute the distance between the point and the plane.
   (a) $(3, 1, -2),\ x + 2y - 2z = 4$
   (b) $(-1, -1, 2),\ 2x + 5y - 6z = 4$

8. Compute the distance between the two parallel lines.
   (a) $2x - y - z = 5$ and $-4x + 2y + 2z = 12$
   (b) $2x - y + z = 1$ and $2x - y + z = -1$

1. Let $V = \mathbb{R}^2$ and for all $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_2, v_2)$, let $\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2)$ and $k\mathbf{u} = (0, ku_2)$ for any scalar k.
(a) Prove that Axioms 7, 8, 9 are true.
(b) Prove that V is not a vector space because Axiom 10 is false.

2. Let $V = \mathbb{R}^2$ and for all $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_2, v_2)$, let $\mathbf{u}+\mathbf{v} = (u_1+v_1+1, u_2+v_2+1)$ and $k\mathbf{u} = (ku_1, ku_2)$ for any scalar k.
(a) Prove that Axioms 4 and 5 are true.
(b) Prove that V is not a vector space.

3. Let $V = \mathbb{R}^2$ and for all $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_2, v_2)$, let $\mathbf{u} + \mathbf{v} = (u_1 + v_1, u_2 + v_2)$ and $k\mathbf{u} = (k^2u_1, k^2u_2)$ for any scalar k. Is V is a vector space? Prove true or false.

4. Prove the subset W is or is not a subspace of $\mathbb{R}^3$.
(a) $W = \{(a, 0, 0) \mid a \in \mathbb{R}\}$
(b) $W = \{(a, 1, 1) \mid a \in \mathbb{R}\}$
(c) $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c\}$
(d) $W = \{(a, b, c) \in \mathbb{R}^3 \mid b = a + c + 1\}$

5. Prove the subset W is or is not a subspace of $M_{2,2}$.
(a) $W = \left\{ \begin{bmatrix} \text{a} & \text{b} \\ \text{c} & \text{–a} \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$
(b) $W = \left\{ \begin{bmatrix} \text{a} & \text{b} \\ \text{b} & \text{d} \end{bmatrix} : a, b, d \in \mathbb{R} \right\}$
(c) $W = \{A \in M_{2,2} \mid A^T = -A\}$
(d) $W = \{A \in M_{2,2} \mid \det A = 0\}$

6. Prove the subset W is or is not a subspace of P_3.
(a) $W = \{a_1x + a_2x^2 + a_3x^3 \mid a_1, a_2, a_3 \in \mathbb{R}\}$
(b) $W = \{a_0 + a_1x + a_2x^2 + a_3x^3 \mid a_0 + a_1 + a_2 + a_3 = 0\}$
(c) $W = \{a_0 + a_1x + a_2x^2 + a_3x^3 \mid a_0, a_1, a_2, a_3 \in \mathbb{Q}\}$
(d) $W = \{a_0 + a_1x \mid a_0, a_1 \in \mathbb{R}\}$

7. Prove the subset W is or is not a subspace of $F(-\infty, \infty)$.
(a) $W = \{f \in F(-\infty, \infty) \mid f(0) = 0\}$
(b) $W = \{f \in F(-\infty, \infty) \mid f(0) = 1\}$
(c) $W = \{f \in F(-\infty, \infty) \mid f(-x) = f(x) \text{ for all } x\}$
(d) $W = \{f \in F(-\infty, \infty) \mid f(x) \leq 0 \text{ for all } x\}$

8. Prove the subset W is or is not a subspace of $\mathbb{R}^\infty$.
(a) $W = \{(v, 0, v, 0, v, 0, \ldots) \mid v \in \mathbb{R}\}$
(b) $W = \{(v, 1, v, 1, v, 1, \ldots) \mid v \in \mathbb{R}\}$
(c) $W = \{(v, 2v, 2^2v, 2^3, 2^4v, \ldots) \mid v \in \mathbb{R}\}$
(d) $W = \{(0, v_1, v_2, v_3, \ldots) \mid v_i \in \mathbb{R}\}$

9. Prove that if W_1 and W_2 are two subspaces of a vector spave V, then $W_1 \cap W_2$ is also a subspave of V.

10. Let $A \in M_{m,n}$ and let $W = \{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = \mathbf{0}\}$. Prove that W is a subspace of $\mathbb{R}^n$.

Homework 10 — Linear Algebra

1. Let $\mathbf{u} = (0, -2, 2)$ and $\mathbf{v} = (1, 3, -1)$. Determine whether or not $\mathbf{x}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$.
   (a) $\mathbf{x} = (2, 2, 2)$ (b) $\mathbf{x} = (0, 4, 5)$ (c) $\mathbf{x} = (0, 0, 0)$

2. Let $\mathbf{u} = (2, 1, 4)$ and $\mathbf{v} = (1, -1, 3)$ and $\mathbf{w} = (3, 2, 5)$. Write $\mathbf{x}$ as a linear combination of $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^3$.
   (a) $\mathbf{x} = (5, 9, 5)$ (b) $\mathbf{x} = (2, 0, 6)$ (c) $\mathbf{x} = (0, 0, 0)$ (d) $\mathbf{x} = (2, 2, 3)$

3. Let $A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$. Determine whether or not M is a linear combination of A, B, and C in $M_{2,2}$.
   (a) $M = \begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$ (b) $M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $M = \begin{bmatrix} -1 & 5 \\ 7 & 1 \end{bmatrix}$

4. Let $p_1 = 2 + x + 4x^2$ and $p_2 = 1 - x + 3x^2$ and $p_3 = 3 + 2x + 5x^2$. Write f as a linear combination of p_1, p_2, and p_3 in P_2.
   (a) $f = 5 + 9x + 5x^2$ (b) $f = 2 + 6x^2$ (c) $f = 0$ (d) $f = 2 + 2x + 3x^2$

5. Determine whether or not $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^3$.
   (a) $\mathbf{v}_1 = (2, 2, 2)$, $\mathbf{v}_2 = (0, 0, 3)$, $\mathbf{v}_3 = (0, 1, 1)$
   (b) $\mathbf{v}_1 = (2, -1, 3)$, $\mathbf{v}_2 = (4, 1, 2)$, $\mathbf{v}_3 = (8, -1, 8)$

6. Let $\mathbf{v}_1 = (2, 1, 0, 3)$ and $\mathbf{v}_2 = (3, -1, 5, 2)$ and $\mathbf{v}_3 = (-1, 0, 2, 1)$. Determine whether or not $\mathbf{x} \in \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbb{R}^4$.
   (a) $\mathbf{x} = (2, 3, -7, 3)$ (b) $\mathbf{x} = (0, 0, 0, 0)$ (c) $\mathbf{x} = (1, 1, 1, 1)$ (d) $\mathbf{x} = (-4, 6, -13, 4)$

7. Let $p_1 = 1 - x + 2x^2$ and $p_2 = 3 + x$ and $p_3 = 5 - x + 4x^2$ and $p_4 = -2 - 2x + 2x^2$. Determine whether or not $\{p_1, p_2, p_3, p_4\}$ span P_2.

Homework 11 — Linear Algebra

1. Determine linearly dependent or independent in $\mathbb{R}^3$.
   (a) $(-1, 2, 4), (5, -10, -20)$
   (b) $(-3, 0, 4), (5, -1, 2), (1, 1, 3)$
   (c) $(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)$

2. Determine linearly dependent or independent in $\mathbb{R}^4$.
   (a) $(3, 8, 7, -3), (1, 5, 3, -1), (2, -1, 2, 6), (4, 2, 6, 4)$
   (b) $(3, 0, -3, 6), (0, 2, 3, 1), (0, -2, -2, 0), (-2, 1, 2, 1)$

3. Determine linearly dependent or independent in P_2.
   (a) $2 - x + 4x^2,\ 3 + 6x + 2x^2,\ 2 + 10x - 4x^2$
   (b) $1 + 3x + 3x^2,\ x + 4x^2,\ 5 + 6x + 3x^2,\ 7 + 2x - x^2$

4. Determine linearly dependent or independent in $M_{m,n}$.
   (a) $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$
   (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

5. Determine linearly dependent or independent in $F(-\infty,\infty)$ using Wronskian.
(a) $x,\ \cos x$ (b) $1,\ x,\ e^x$ (c) $e^x,\ xe^x,\ x^2e^x$ (d) $\sin x,\ \cos x,\ x\cos x$

6. Let $\mathbf{u} = (k, -\frac{1}{2}, -\frac{1}{2})$ and $\mathbf{v} = (-\frac{1}{2}, k, -\frac{1}{2})$ and $\mathbf{w} = (-\frac{1}{2}, -\frac{1}{2}, k)$. Determine all values of k such that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ become linearly dependent in $\mathbb{R}^3$.

7. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & k \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ k & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$. Determine all values of k such that $\{A, B, C\}$ become linearly independent in $M_{2,2}$.

8. Show that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are linearly dependent in $\mathbb{R}^4$ by writing each vector as a linear combination of the other two.
(a) $\mathbf{u} = (0, 3, 1, -1), \mathbf{v} = (6, 0, 5, 1), \mathbf{w} = (4, -7, 1, 3)$
(b) $\mathbf{u} = (1, 2, 3, 4), \mathbf{v} = (0, 1, 0, -1), \mathbf{w} = (1, 3, 3, 3)$

9. Determine whether or not the three vectors in $\mathbb{R}^3$ lie on the same line.
(a) $(3, -6, 9), (2, -4, 6), (1, 1, 1)$
(b) $(2, -1, 4), (4, 2, 3), (2, 7, -6)$
(c) $(4, 6, 8), (2, 3, 4), (-2, -3, -4)$

10. Determine whether or not the three vectors in $\mathbb{R}^3$ lie in a plane.
(a) $(2, -2, 0), (6, 1, 4), (2, 0, -4)$
(b) $(-6, 7, 2), (3, 2, 4), (4, -1, 2)$

Homework 12 — Linear Algebra

1. Determine a basis or not a basis for $\mathbb{R}^2$.
(a) $\{(2, 1), (3, 0)\}$ (b) $\{(1, 2), (0, 3), (0, 5)\}$

2. Determine a basis or not a basis for $\mathbb{R}^3$.
(a) $\{(-1, 3, 2), (6, 1, 1)\}$
(b) $\{(3, 1, -4), (2, 5, 6), (1, 4, 8)\}$
(c) $\{(2, -3, 1), (4, 1, 1), (0, -7, 1)\}$
(d) $\{(1, 6, 4), (2, 4, -1), (-1, 2, 5)\}$

3. Determine a basis or not a basis for P_2.
(a) $\{x, 1 + x + x^2\}$
(b) $\{x^2 + 1, x^2 - 1, 2x - 1\}$
(c) $\{1 - 3x + 2x^2, 1 + x + 4x^2, 1 - 7x\}$

4. Determine a basis or not a basis for P_3.
(a) $\{x, 1 - x, 1 + x - x^3\}$
(b) $\{1 + x, 1 - x, 1 - x^2, 1 - x^3\}$
(c) $\{1, 2x, -2 + 4x^2, -12x + 8x^3\}$

5. Determine a basis or not a basis for $M_{2,2}$.
(a) $\left\{ \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \right\}$
(b) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$
(c) $\left\{ \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \right\}$

6. Find the coordinate vector of $\mathbf{w}$ relative to the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for $\mathbb{R}^2$.
(a) $\mathbf{u}_1 = (2, -4), \mathbf{u}_2 = (3, 8), \mathbf{w} = (1, 1)$
(b) $\mathbf{u}_1 = (1, 1), \mathbf{u}_2 = (0, 2), \mathbf{w} = (a, b)$
(c) $\mathbf{u}_1 = (1, -1), \mathbf{u}_2 = (1, 1), \mathbf{w} = (1, 0)$
(d) $\mathbf{u}_1 = (1, -1), \mathbf{u}_2 = (1, 1), \mathbf{w} = (0, 1)$

7. Find the coordinate vector of $\mathbf{w}$ relative to the basis $\{\mathbf{u}_1, \mathbf{u}_2\, \mathbf{u}_3\}$ for $\mathbb{R}^3$.
(a) $\mathbf{u}_1 = (1, 0, 0), \mathbf{u}_2 = (2, 2, 0), \mathbf{u}_3 = (3, 3, 3), \mathbf{w} = (2, -1, 3)$
(b) $\mathbf{u}_1 = (1, 2, 3), \mathbf{u}_2 = (-4, 5, 6), \mathbf{u}_3 = (7, -8, 9), \mathbf{w} = (5, -12, 3)$

8. Find the coordinate vector of p relative to the basis $\{p_1, p_2\, p_3\}$ for P_2.
(a) $p_1 = 1, p_2 = x, p_3 = x^2, p = 4 - 3x + x^2$
(b) $p_1 = 1 + x, p_2 = 1 + x^2, p_3 = x + x^2, p = 2 - x + x^2$
(c) $p_1 = 1 + x + x^2, p_2 = x + x^2, p_3 = x^2, p = 7 - x + 2x^2$
(d) $p_1 = 1 + 2x + x^2, p_2 = 2 + 9x, p_3 = 3 + 3x + 4x^2, p = 2 + 17x - 3x^2$

9. Find the coordinate vector of M relative to the basis $\{A, B, C, D\}$ for $M_{2,2}$.
(a) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, M = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$
(b) $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, M = \begin{bmatrix} 6 & 2 \\ 5 & 3 \end{bmatrix}$

Homework 13 — Linear Algebra

1. Find the dimension and a basis for the solution space of $A\mathbf{x} = \mathbf{0}$ in $\mathbb{R}^3$.
(a) $A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$
(c) $A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -6 & 2 \\ 3 & -9 & 3 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -2 \\ 4 & 3 & -1 \\ 6 & 5 & 1 \end{bmatrix}$

2. Find the dimension and a basis for the solution space of $A\mathbf{x} = \mathbf{0}$ in $\mathbb{R}^4$.
(a) $A = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 5 & -1 & 1 & -1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & -4 & 3 & -1 \\ 2 & -8 & 6 & -2 \end{bmatrix}$

3. Find the dimension and a basis for the given subspace of $\mathbb{R}^3$.
(a) The plane $3x - 2y + 5z = 0$.
(b) The plane $x - y = 0$.
(c) The line $x = 2t, y = -t, z = 4t$.
(d) All vectors (a, b, c) with $b = a + c$.

4. Find the dimension and a basis for the given subspace of $\mathbb{R}^4$.
(a) All vectors of the form $(a, b, c, 0)$.
(b) All vectors (a, b, c, d) with $d = a + b$ and $c = a - b$.
(c) All vectors of the form (a, a, a, a).

5. Find the dimension and a basis for the given subspace of $M_{3,3}$.
(a) All diagonal 3×3 matrices.
(b) All symmetric 3×3 matrices.
(c) All upper triangular 3×3 matrices.

6. Find a standard basis vector to be added to $\{\mathbf{v}_1, \mathbf{v}_2\}$ to make a basis for $\mathbb{R}^3$.
(a) $\mathbf{v}_1 = (-1, 2, 3), \mathbf{v}_2 = (1, -2, -2)$
(b) $\mathbf{v}_1 = (1, -1, 0), \mathbf{v}_2 = (3, 1, -2)$
(c) $\mathbf{v}_1 = (1, -2, 3), \mathbf{v}_2 = (0, 5, -3)$

7. Find two standard basis vectors to be added to $\{\mathbf{v}_1, \mathbf{v}_2\}$ to make a basis for $\mathbb{R}^4$.
(a) $\mathbf{v}_1 = (1, -4, 2, -3), \mathbf{v}_2 = (-3, 8, -4, 6)$
(b) $\mathbf{v}_1 = (1, 0, 0, 0), \mathbf{v}_2 = (1, 1, 0, 0)$

8. Find a basis for the subspace $W = \text{span}\{(1, 0, 0), (1, 0, 1), (2, 0, 1), (0, 0, -1)\}$ in $\mathbb{R}^3$.

9. Find a basis for the subspace spanned by $\{(1, 1, 1, 1), (2, 2, 2, 0), (0, 0, 0, 3), (3, 3, 3, 4)\}$ in $\mathbb{R}^4$.

10. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for a vector space V, and let $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_1 + \mathbf{v}_2$ and $\mathbf{u}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$. Prove that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is also a basis for V.

Homework 14 — Linear Algebra

1. Given two bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2\}$ for $\mathbb{R}^2$, find (1) the transition matrix from B' to B (2) the transition matrix from B to B' (3) the coordinate vector of $\mathbf{w}$ relative to B (4) the coordinate vector of $\mathbf{w}$ relative to B'.
(a) $\mathbf{u}_1 = (2, 2), \mathbf{u}_2 = (4, -1), \mathbf{u}'_1 = (1, 3), \mathbf{u}'_2 = (-1, -1), \mathbf{w} = (3, -5)$
(b) $\mathbf{u}_1 = (1, 0), \mathbf{u}_2 = (0, 1), \mathbf{u}'_1 = (2, 1), \mathbf{u}'_2 = (-3, 4), \mathbf{w} = (3, -5)$
(c) $\mathbf{u}_1 = (1, 2), \mathbf{u}_2 = (2, 3), \mathbf{u}'_1 = (1, 3), \mathbf{u}'_2 = (1, 4), \mathbf{w} = (0, 1)$
(d) $\mathbf{u}_1 = (2, 1), \mathbf{u}_2 = (-3, 4), \mathbf{u}'_1 = (1, 0), \mathbf{u}'_2 = (0, 1), \mathbf{w} = (5, -3)$

2. Given two bases $B = \{\mathbf{u}_1, \mathbf{u}_2\, \mathbf{u}_3\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2, \mathbf{u}'_3\}$ for $\mathbb{R}^3$, find (1) the transition matrix $P_{B \to B'}$ (2) the coordinate vector $[\mathbf{w}]_B$ (3) the coordinate vector $[\mathbf{w}]_{B'}$.
(a) $\mathbf{u}_1 = (2, 1, 1), \mathbf{u}_2 = (2, -1, 1), \mathbf{u}_3 = (1, 2, 1), \mathbf{u}'_1 = (3, 1, -5), \mathbf{u}'_2 = (1, 1, -3), \mathbf{u}'_3 = (-1, 0, 2), \mathbf{w} = (-5, 8, -5)$
(b) $\mathbf{u}_1 = (-3, 0, -3), \mathbf{u}_2 = (-3, 2, -1), \mathbf{u}_3 = (1, 6, -1), \mathbf{u}'_1 = (-6, -6, 0), \mathbf{u}'_2 = (-2, -6, 4), \mathbf{u}'_3 = (-2, -3, 7), \mathbf{w} = (-5, 8, -5)$
(c) $\mathbf{u}_1 = (1, 0, 0), \mathbf{u}_2 = (0, 1, 0), \mathbf{u}_3 = (0, 0, 1), \mathbf{u}'_1 = (1, 2, 1), \mathbf{u}'_2 = (2, 5, 0), \mathbf{u}'_3 = (3, 3, 8), \mathbf{w} = (5, -3, 1)$

3. Fix two bases $B = \{6 + 3x, 10 + 2x\}$ and $B' = \{2, 3 + 2x\}$ for P_1. Let $p = -4 + x$.
(a) Find the transition matrix from B' to B.
(b) Find the transition matrix from B to B'.
(c) Find the coordinate vector of p relative to B.
(d) Find the coordinate vector of p relative to B'.

4. Let $W = \text{span}\{\sin x, \cos x\}$, a subspace of $F(-\infty, \infty)$, and let $B = \{\sin x, \cos x\}$ be a basis for W. Let $g_1 = 2 \sin x + \cos x$ and $g_2 = 3 \cos x$.
(a) Prove that $B' = \{g_1, g_2\}$ is a basis for W.

(b) Find the transition matrix $P_{B'\to B}$.
(c) Find the transition matrix $P_{B\to B'}$.
(d) Find the coordinate vectors $[f]_B$ and $[f]_{B'}$, where $f = 2\sin x - 5\cos x$.

5. Fix three bases B_1, B_2, B_3 for $\mathbb{R}^2$ with $P_{B_1\to B_2} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ and $P_{B_2\to B_3} = \begin{bmatrix} 7 & 2 \\ 4 & -1 \end{bmatrix}$.
   (a) Find the transition matrix $P_{B_1\to B_3}$.
   (b) Find the transition matrix $P_{B_3\to B_1}$.

6. Let $P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$.
   (a) Find the basis B for $\mathbb{R}^3$ such that $P_{B\to S} = P$.
   (b) Find the basis B for $\mathbb{R}^3$ such that $P_{S\to B} = P$.

7. Let $B = \{(1,1,1), (1,1,0), (1,0,0)\}$ be a basis for $\mathbb{R}^3$. Find another basis B' such that $P_{B'\to B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$.

Homework 15 — Linear Algebra

1. Write **b** as a linear combination of the column vectors of A, when possible.
   (a) $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 1 & 3 \\ 0 & 1 & 2 & 2 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 5 \\ 7 \end{bmatrix}$

2. Solve the system of linear equations using vector form.
   (a) $\begin{cases} x - 3y = 1 \\ 2x - 6y = 2 \end{cases}$ (b) $\begin{cases} x + y + 2z = 5 \\ x + z = -2 \\ 2x + y + 3z = 3 \end{cases}$
   (c) $\begin{cases} x - 2y + z + 2w = -1 \\ 2x - 4y + 2z + 4w = -2 \\ -x + 2y - z - 2w = 1 \\ 3x - 6y + 3z + 6w = -3 \end{cases}$ (d) $\begin{cases} x + 2y - 3z + w = 4 \\ -2x + y + 2z + w = -1 \\ -x + 3y - z + 2w = 3 \\ 4x - 7y - 5w = -5 \end{cases}$

3. Find a basis for the null space and row space of A.
   (a) $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$

4. Let $A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$.
   (a) Find a basis for the row space of A.
   (b) Find a basis for the column space of A.
   (c) Find a basis for the row space of A from its row vectors.

5. Find a basis for the row space of A from its row vectors.
   (a) The matrix A in Problem 3(c).
   (b) The matrix A in Problem 3(d).

6. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the given vectors.
   (a) $\{(1,1,-4,-3),(2,0,2,-2),(2,-1,3,2),(0,-1,1,4)\}$
   (b) $\}(1,1,0,0),(0,0,1,1),(-2,0,2,2),(0,-3,0,3)\}$

7. Let $\mathbf{v}_1 = (1,-1,5,2), \mathbf{v}_2 = (-2,3,1,0), \mathbf{v}_3 = (4,-5,9,4).\mathbf{v}_4 = (0,4,2,-3), \mathbf{v}_5 = (-7,18,2,-8)$. Find a basis for the subspace span$\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4,\mathbf{v}_5\}$ using the same vectors, and write the remaining vectors as a linear combination of the basis vectors.

Homework 16 — Linear Algebra

1. Determine the rank and nullity of the matrix given by its reduced row echelon form.
   (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. Use row echelon form to determine the rank and nullity of the matrix A.
   (a) $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -3 & 3 \\ 4 & 8 & -4 & 4 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & -2 & 2 & 3 & 1 \\ -3 & 6 & -1 & 1 & -7 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$
   (c) $A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 1 & 3 & 0 & -4 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & -2 \end{bmatrix}$

3. Suppose that the matrix A is 5×9 and the column space has dimension 3.
   (a) Determine the rank of A.
   (b) Determine the nullity of A.
   (c) Determine the rank of A^T.
   (d) Determine the nullity of A^T.

4. Suppose that the matrix A is 10×6 and the row space has dimension 6.
   (a) Determine the rank of A.
   (b) Determine the nullity of A.
   (c) Determine the rank of A^T.
   (d) Determine the nullity of A^T.

5. Suppose that the matrix A is 4×6.
(a) Determine the possible values for the rank of A.
(b) Determine the possible values for the nullity of A.
(c) Determine the possible values for the rank of A^T.
(d) Determine the possible values for the nullity of A^T.

Homework 17 Linear Algebra

1. Use matrix multiplication to find the result of the transformation on the given vector.
(a) Reflection of $(-1, 2)$ about the line $y = x$.
(b) Reflection of $(2, -5, 3)$ about the xy-plane.
(c) Orthogonal projection of $(2, -5)$ onto the x-axis.
(d) Orthogonal projection of $(-2, 1, 3)$ onto the yz-plane.
(e) Rotation of $(3, -4)$, 45^o about the origin.
(f) Contraction of $(-1, 2)$ with factor $k = \frac{1}{2}$.
(g) Dilation of $(2, -1, 3)$ with factor $k = 2$.
(h) Compression of $(-1, 2)$ in the x-direction with factor $k = \frac{1}{2}$.

2. Find the standard matrix for the composition of transformations in $\mathbb{R}^2$.
(a) Rotation of 90^o followed by reflection about the line $y = x$.
(b) Orthogonal projection onto the y-axis followed by contraction with factor $k = \frac{1}{2}$.
(c) Reflection about the x-axis followed by dilation with factor $k = 3$, then followed by rotation of 60^o about the origin.
(d) Rotation of 60^o about the origin followed by orthogonal projection onto the x-axis, then followed by reflection about the line $y = x$.

3. Let $T_1(x, y) = (x + y, x - y)$ and $T_2(x, y) = (3x, 2x + 4y)$. Find the formulas for $T_1(T_2(x, y))$ and $T_2(T_1(x, y))$ using matrix multiplication.

4. Let $T_1(x, y, z) = (4x, -2x + y, -x - 3y)$ and $T_2(x, y, z) = (x + 2y, -z, 4x - z)$. Find the formulas for $T_1(T_2(x, y, z))$ and $T_2(T_1(x, y, z))$ using matrix multiplication.

5. Determine whether or not $T(x, y) = (w_1, w_2)$ is one-to-one, and if so, find $T^{-1}(w_1, w_2)$.
(a) $\begin{cases} w_1 = 8x + 4y \\ w_2 = 2x + y \end{cases}$ (b) $\begin{cases} w_1 = 2x - 3y \\ w_2 = 5x + y \end{cases}$

(c) $\begin{cases} w_1 = x + 2y \\ w_2 = -x + y \end{cases}$ (d) $\begin{cases} w_1 = 4x - 6y \\ w_2 = -2x + 3y \end{cases}$

6. Determine if $T(x, y, z) = (w_1, w_2, w_3)$ is one-to-one, and if so, find $T^{-1}(w_1, w_2, w_3)$.
(a) $\begin{cases} w_1 = -x + 3y + 2z \\ w_2 = 2x + 4z \\ w_3 = x + 3y + 6z \end{cases}$ (b) $\begin{cases} w_1 = x + 2y + 3z \\ w_2 = 2x + 5y + 3z \\ w_3 = x + 8z \end{cases}$

(c) $\begin{cases} w_1 = x - 2y + 2z \\ w_2 = 2x + y + z \\ w_3 = x + y \end{cases}$ (d) $\begin{cases} w_1 = x - 3y + 4z \\ w_2 = -x + y + z \\ w_3 = -2y + 5z \end{cases}$

7. Use kernel to determine whether or not the transformation $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one.

(a) $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & -4 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & -4 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

Homework 18 — Linear Algebra

1. Given the eigenvector $\mathbf{x}$ for the matrix A, find the corresponding eigenvalue.

(a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ (d) $A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

2. Find the characteristic equation of the triangular matrix A.

(a) $A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 9 & -8 & 6 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

3. Find each eigenvalue and a basis for the eigenspace of A.

(a) $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $A = \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$ (c) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(e) $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (f) $A = \begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ (g) $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (h) $A = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$

4. Find each eigenvalue and a basis for the eigenspace of A.

(a) $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix}$ (c) $A = \begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ (e) $A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (f) $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

5. Find the eigenvalues and a basis for the eigenspace of the linear operator.
(a) $T(x, y) = (x + 4y, 2x + 3y)$
(b) $T(x, y, z) = (2x - y - z, x - z, -x + y + 2z)$

6. Define $D : C^\infty(-\infty, \infty) \to C^\infty(-\infty, \infty)$ by $D(f) = f''$.
(a) Show that D is a linear operator.
(b) For every $\omega > 0$, show that $\sin \omega x$ and $\cos \omega x$ are eigenvectors of D, and find the corresponding eigenvalues.

7. Find a 3×3 matrix A that has eigenvalues 1, -1, and 0, with corresponding eigenvectors $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

Homework 19 — Linear Algebra

1. Find a matrix P that diagonalizes A, and verify that $P^{-1}AP$ is diagonal.

   (a) $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$ (b) $A = \begin{bmatrix} -14 & 12 \\ -20 & 17 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

2. Use diagonalization to compute the matrix A^{10}.

   (a) $A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ (c) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

3. Given that $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$ diagonalizes $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$, compute A^{11}.

4. Given that $P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ diagonalizes $A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, compute A^k for (a) $k = 1000$ (b) $k = -1000$ (c) $k = 2301$ (d) $k = -2301$.

5. Given $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$, compute A^n for an arbitrary $n \in \mathbb{N}$.

6. Prove that if A is similar to B and B is similar to C, then A is similar to C.

7. Prove that if A is similar to B, then $\det A = \det B$.

8. Use contradiction to show that the two matrices A and B are not similar.

   (a) $A = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & -2 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9. Show that $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are similar.

Homework 20 — Linear Algebra

1. Let $\mathbb{R}^2$ have the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2$. Let $\mathbf{u} = (1, 1)$ and $\mathbf{v} = (3, 2)$ and $\mathbf{w} = (0, -1)$, then compute
   (a) $\langle \mathbf{u}, \mathbf{v} \rangle$ (b) $\langle 3\mathbf{v}, \mathbf{w} \rangle$ (c) $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle$ (d) $\|\mathbf{v}\|$ (e) $d(\mathbf{u}, \mathbf{v})$ (f) $\|\mathbf{u} - 3\mathbf{v}\|$.

2. Repeat Problem (1) using $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{2}u_1v_1 + 5u_2v_2$.

3. Let $\langle \mathbf{u}, \mathbf{v} \rangle = 2$, $\langle \mathbf{v}, \mathbf{w} \rangle = -6$, $\langle \mathbf{u}, \mathbf{w} \rangle = -3$ and $\|\mathbf{u}\| = 1$, $\|\mathbf{v}\| = 2$, $\|\mathbf{w}\| = 7$. Find
   (a) $\langle 2\mathbf{v} - \mathbf{w}, 3\mathbf{u} + 2\mathbf{w} \rangle$ (b) $\langle \mathbf{u} - \mathbf{v} - 2\mathbf{w}, 4\mathbf{u} + \mathbf{v} \rangle$ (c) $\|\mathbf{u} + \mathbf{v}\|$ (d) $\|2\mathbf{w} - \mathbf{v}\|$

4. Compute $\langle (0, -3), (6, 2) \rangle$ using the inner product on $\mathbb{R}^2$ generated by the matrix A.
   (a) $A = \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}$ (b) $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

5. Let $\mathbf{u} = (-1, 2)$ and $\mathbf{v} = (2, 5)$. Compute $\|\mathbf{u}\|$ and $d(\mathbf{u}, \mathbf{v})$ using the inner product on $\mathbb{R}^2$ generated by the matrix A.
   (a) $A = \begin{bmatrix} 4 & 0 \\ 3 & 5 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

6. Compute $\langle A, B \rangle$ using the standard inner product on $M_{2,2}$.
   (a) $A = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 6 \\ 0 & 8 \end{bmatrix}$

7. Continue Problem (6) and compute $\|A\|$ and $d(A, B)$.

8. Compute $\langle p, q \rangle$ using the standard inner product on P_2.
   (a) $p = -2 + x + 3x^2$, $q = 4 - 7x^2$
   (b) $p = -5 + 2x + x^2$, $q = 3 + 2x - 4x^2$

9. Continue Problem (8) and compute $\|p\|$ and $d(p, q)$.

Homework 21 — Linear Algebra

1. Use the Euclidean inner product to find the cosine of the angle between two vectors.
   (a) $\mathbf{u} = (1, -3)$, $\mathbf{v} = (2, 4)$
   (b) $\mathbf{u} = (-1, 5, 2)$, $\mathbf{v} = (2, 4, -9)$
   (c) $\mathbf{u} = (1, 0.1.0)$, $\mathbf{v} = (-3, -3, -3, -3)$

2. Use the standard inner product to find the cosine of the angle between $p, q \in P_2$.
   (a) $p = -1 + 5x + 2x^2$, $q = 2 + 4x - 9x^2$
   (b) $p = x - x^2$, $q = 7 + 3 + 3x^2$

3. Use the standard inner product to find the cosine of the angle between $A, B \in M_{2,2}$.
   (a) $A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}$

4. Find a value of k such that the vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal with respect to the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + ku_2v_2$.
   (a) $\mathbf{u} = (1, 3), \mathbf{v} = (2, -1)$ (b) $\mathbf{u} = (2, -4), \mathbf{v} = (0, 3)$

5. Verify the Cauchy-Schwarz inequality for the given pair of vectors.
   (a) $\mathbf{u} = (1, 0, 3), \mathbf{v} = (2, 1, -1) \in \mathbb{R}^3$ with inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2 + u_3v_3$.
   (b) $A = \begin{bmatrix} -1 & 2 \\ 6 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix} \in M_{2,2}$ with the standard inner product.
   (c) $p = -1 + 2x + x^2$, $q = 2 - 4x^2 \in P_2$ with the standard inner product.

6. Let $p = x$ and $q = x^2 \in C[0,1]$ with the integral inner product $\langle p, q\rangle = \int_0^1 p(x)q(x)dx$.
(a) Compute $\langle p, q\rangle$.
(b) Compute $\|p\|$ and $\|q\|$.
(c) Compute the cosine of the angle between p and q.
(d) Compute $d(p, q)$.

7. Repeat Problem (6) with $p = x^2 - x,\ q = x + 1 \in C[-1,1]$ and $\langle p, q\rangle = \int_{-1}^1 p(x)q(x)dx$.

8. Let V be an inner product space. Prove that if $\mathbf{u}, \mathbf{v} \in V$ are orthogonal unit vector, then $\|\mathbf{u} - \mathbf{v}\| = \sqrt{2}$.

9. Let V be an inner product space. Prove that if $\mathbf{u} \in V$ is orthogonal to $\mathbf{v}_1 \in V$ and to $\mathbf{v}_2 \in V$, then $\mathbf{u}$ is orthogonal to all $\mathbf{w} \in \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

Homework 22 — Linear Algebra

1. Verify that $\mathbf{v}_1 = (-\frac{3}{5}, \frac{4}{5}, 0), \mathbf{v}_2 = (\frac{4}{5}, \frac{3}{5}, 0), \mathbf{v}_3 = (0, 0, 1)$ form an orthonormal basis for $\mathbb{R}^3$ with respect to the Euclidean inner product, then write the vector $\mathbf{u}$ as their linear combination.
(a) $\mathbf{u} = (1, -2, 2)$ (b) $\mathbf{u} = (3, -7, 4)$

2. Verify that $\mathbf{v}_1 = (2, -2, 1), \mathbf{v}_2 = (2, 1, -2), \mathbf{v}_3 = (1, 2, 2)$ form an orthogonal basis for $\mathbb{R}^3$ with respect to the Euclidean inner product, then write the vector $\mathbf{u} = (-1, 0, 2)$ as their linear combination.

3. Verify that $\mathbf{v}_1 = (1, -1, 2, -1), \mathbf{v}_2 = (-2, 2, 3, 2), \mathbf{v}_3 = (1, 2, 0, -1), \mathbf{v}_4 = (1, 0, 0, 1)$ form an orthogonal basis for $\mathbb{R}^4$ with respect to the Euclidean inner product, then write the vector $\mathbf{u} = (1, 1, 1, 1)$ as their linear combination.

4. Verify that $p_1 = 1, p_2 = x, p_3 = \frac{3}{2}x^2 - \frac{1}{2}$ form an orthogonal basis for P_2 with respect to the inner product $\langle p, q\rangle = \int_{-1}^1 p(x)q(x)dx$, then write the polynomial p as their linear combination.
(a) $p = 1 + x + 4x^2$ (b) $p = 2 - 7x^2$ (c) $p = 4 + 3x$

5. Use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for $\mathbb{R}^2$ into an orthonormal basis with respect to the Euclidean inner product.
(a) $\mathbf{u}_1 = (1, -3), \mathbf{u}_2 = (2, 2)$
(b) $\mathbf{u}_1 = (1, 0), \mathbf{u}_2 = (3, -5)$

6. Use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for $\mathbb{R}^3$ into an orthonormal basis with respect to the Euclidean inner product.
(a) $\mathbf{u}_1 = (1, 1, 1), \mathbf{u}_2 = (-1, 1, 0), \mathbf{u}_3 = (1, 2, 1)$
(b) $\mathbf{u}_1 = (1, 0, 0), \mathbf{u}_2 = (3, 7, -2), \mathbf{u}_3 = (0, 4, 1)$

7. Transform the basis $\{(0, 2, 1, 0), (1, -1, 0, 0), (1, 2, 0, -1), (1, 0, 0, 1)\}$ for $\mathbb{R}^4$ into an orthonormal basis with respect to the Euclidean inner product.

8. Transform the basis $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ for $\mathbb{R}^3$ into an orthonormal basis with respect to the inner product $\langle \mathbf{u}, \mathbf{v}\rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$.

9. Transform the standard basis $\{1, x, x^2\}$ for P_2 into an orthonormal basis with respect to the inner product $\langle p, q\rangle = \int_0^1 p(x)q(x)dx$.

Homework 1

1. (a) $(2, -3)$ (b) $(-1, \frac{1}{2})$ (c) $(-8, -4)$
2. (a) $(4, 3, 2)$
   (b) $(2 - 3t, 4 + t, 2 - t, t)$
   (c) $\emptyset$
   (d) $(-1 - 5s - 5t, s, 1 - 3t, 2 - 4t, t)$
3. (a) $(3, 1, 2)$
   (b) $(\frac{2}{3}, 0, -\frac{1}{3})$
   (c) $(1, 1, -1, -1)$
   (d) $(-\frac{s}{4}, -\frac{s}{4} - t, s, t)$
4. See Problem (3).
5. (a) $k = -4$ (b) $k \neq \pm 4$ (c) $k = 4$

Homework 2

1. $(5, -3, 4, 1)$
2. (a) $(67, 41, 41)$ (b) $(63, 67, 57)$
   (c) $(41, 21, 67)$ (d) $(6, 6, 63)$
   (e) $(24, 56, 97)$ (f) $(76, 98, 97)$
3. (a) $\begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix}$
   (c) $\begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & 8 & 19 \\ -2 & 0 & 0 \\ 32 & 9 & 25 \end{bmatrix}$
   (e) $\begin{bmatrix} 14 & 36 & 25 \\ 4 & -1 & 7 \\ 12 & 26 & 21 \end{bmatrix}$ (f) $\begin{bmatrix} 5 & 9 \\ 13 & 9 \end{bmatrix}$
4. (a) $\emptyset$ (b) $\begin{bmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{bmatrix}$
   (c) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$
   (e) $\emptyset$ (f) $\begin{bmatrix} 48 & 15 & 31 \\ 0 & 2 & 6 \\ 38 & 10 & 27 \end{bmatrix}$
5. (a) $\begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
   (e) $\begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}$ (f) $\begin{bmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{bmatrix}$
6. (a) 5 (b) 8 (c) 8 (d) $\emptyset$ (e) -25 (f) 46

Homework 3

1. (a) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}$
   (c) $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{1}{2} & -2 \\ 1 & 3 \end{bmatrix}$
2. (a) $(-5, 14)$ (b) $(3, -4)$ (c) $(\frac{1}{2}, \frac{1}{2})$ (d) $(4, -6)$
3. See Problem (1).
4. (a) $\begin{bmatrix} \frac{3}{2} & \frac{-11}{10} & \frac{-6}{5} \\ -1 & 1 & 1 \\ \frac{-1}{2} & \frac{7}{10} & \frac{2}{5} \end{bmatrix}$
   (b) $\emptyset$
   (c) $\frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$
   (d) $\frac{1}{8}\begin{bmatrix} 8 & 0 & 0 & 0 \\ -4 & 4 & 0 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$
   (e) $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & -3 & 0 \\ \frac{-1}{8} & \frac{1}{4} & \frac{-3}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{40} & \frac{-1}{20} & \frac{-1}{10} & \frac{-1}{5} \end{bmatrix}$
   (f) $\begin{bmatrix} \frac{7}{4} & \frac{1}{2} & -3 & 0 \\ \frac{1}{8} & \frac{1}{4} & \frac{-3}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{-1}{40} & \frac{-1}{20} & \frac{-1}{10} & \frac{-1}{5} \end{bmatrix}$
   (g) $\begin{bmatrix} \frac{-4}{5} & \frac{3}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{3}{2} & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ \frac{4}{5} & \frac{2}{5} & \frac{-1}{5} & \frac{-1}{5} \end{bmatrix}$
5. (a) $(-7, 4, -1)$ (b) $(1, -11, 16)$
   (c) $(-\frac{1}{2}, -\frac{1}{2}, \frac{7}{2})$ (d) $(1, -6, 10, -7)$
6. (a) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$
   (b) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
   (d) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times$
   $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Homework 4

1. (a) 5 (b) 0 (c) 59 (d) $-3\sqrt{6}$

2. (a) 0 (b) 425 (c) 104 (d) -123

3. See Problem (2c).

4. See Problem (2d).

5. (a) 275 (b) -240 (c) -2

Homework 5

1. (a) -21 (b) -5 (c) -7 (d) 18

2. (a) 6 (b) -21 (c) -2

3. (a) 5 (b) 10 (c) -5 (d)10

4. (a) $(\frac{3}{11}, \frac{2}{11}, -\frac{1}{11})$ (b) $(\frac{26}{21}, \frac{25}{21}, \frac{5}{7})$
   (c) $(1, 2)$ (d) $(3, 5, -1, 8)$

Homework 6

1. (a) Yes (b) No (c) No (d) No

2. (a) $k \neq \frac{5\pm\sqrt{17}}{2}$ (b) $k \neq \pm 2$
   (c) $k \neq -1$ (d) $k \neq \frac{1}{4}$

3. (a) $\begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & \frac{3}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -1 & 0 & -1 \end{bmatrix}$
   (c) $\frac{1}{2}\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -4 & 1 & 0 \\ \frac{29}{12} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix}$

Homework 7

1. (a) $(1, -4)$ (b) $(9, 14)$ (c) $(38, 28)$ (d) $(4, 29)$

2. (a) $(-2, 0, 4)$ (b) $(-30, -7, 5)$
   (c) $(-39, 69, -12)$ (d) $(0, -10, 0)$

3. (a) $\sqrt{83}$ (b) $\sqrt{17}+\sqrt{26}$ (c) $\sqrt{454}$ (d) $\sqrt{183}$

4. (a) -10, 5, 100 (b) -34, 73, 84
   (c) 0, 54, 21 (d) -8, 15, 27

5. (a) $5\sqrt{5}; \frac{-1}{\sqrt{5}}$ (b) $\sqrt{59}; 0$
   (c) $\sqrt{54}; \frac{8}{9\sqrt{5}}$ (d) $\sqrt{46}; \frac{1}{2\sqrt{119}}$

6. (a) -3 (b) $4\sqrt{14}$ (c) $-\sqrt{14}$ (d) $-\sqrt{14}$

Homework 8

1. (a) No (b) Yes

2. (a) $-2(x+1)+(y-3)-(z+2)=0$
   (b) $(x-1)+9(y-1)+8(z-4)=0$
   (c) $2z=0$ (d) $x+2y+3z=0$

3. (a) No (b) Yes (c) Yes (d) Yes

4. (a) No (b) Yes

5. (a) $\frac{2}{5}$ (b) $\frac{18}{\sqrt{22}}$ (c) $\frac{43}{\sqrt{54}}$ (d) $\frac{1}{\sqrt{10}}$

6. (a) 1 (b) $\frac{11}{\sqrt{10}}$ (c) $\frac{1}{\sqrt{17}}$ (d) $\frac{6}{\sqrt{10}}$

7. (a) $\frac{5}{3}$ (b) $\frac{23}{\sqrt{65}}$

8. (a) $\frac{11}{\sqrt{6}}$ (b) $\frac{2}{\sqrt{6}}$

Homework 9

1. Proof

2. Axioms 7 and 8 are false.

3. Axioms 7 and 8 are false.

4. (a) Yes (b) No (c) Yes (d) No

5. (a) Yes (b) Yes (c) Yes (d) No

6. (a) Yes (b) Yes (c) No (d) Yes

7. (a) Yes (b) No (c) Yes (d) No

8. (a) Yes (b) No (c) Yes (d) Yes

Homework 10

1. (a) Yes (b) No (c) Yes

2. (a) $3\mathbf{u}-4\mathbf{v}+\mathbf{w}$ (b) $4\mathbf{u}-2\mathbf{w}$
   (c) $0\mathbf{u}+0\mathbf{v}+0\mathbf{w}$ (d) $\frac{1}{2}\mathbf{u}-\frac{1}{2}\mathbf{v}+\frac{1}{2}\mathbf{w}$

3. (a) Yes (b) Yes (c) No

4. (a) $3p_1-4p_2+p_3$ (b) $4p_1-2p_3$
   (c) $0p_1+0p_2+0p_3$ (d) $\frac{1}{2}p_1-\frac{1}{2}p_2+\frac{1}{2}p_3$

5. (a) Yes (b) No

6. (a) Yes (b) Yes (c) No (d) Yes

7. No

Homework 11

1. (a) LD (b) LI (c) LD

2. (a) LD (b) LI

3. (a) LI (b) LD

4. (a) LI (b) LI

5. (a) LI (b) LI (c) LI (d) LI

6. $k \in \{-\frac{1}{2}, 1\}$

7. $k \notin \{-1, 2\}$

8. (a) $\mathbf{w} = -\frac{7}{3}\mathbf{u} + \frac{2}{3}\mathbf{v}$ (b) $\mathbf{w} = \mathbf{u} + \mathbf{v}$

9. (a) No (b) No (c) Yes

10. (a) No (b) Yes

Homework 12

1. (a) Yes (b) No

2. (a) No (b) Yes (c) No (d) No

3. (a) No (b) Yes (c) No

4. (a) No (b) Yes (c) Yes

5. (a) No (b) Yes (c) Yes

6. (a) $(\frac{5}{28}, \frac{3}{14})$ (b) $(a, \frac{b-a}{2})$ (c) $(\frac{1}{2}, \frac{1}{2})$ (d) $(-\frac{1}{2}, \frac{1}{2})$

7. (a) $(3, -2, 1)$ (b) $(-2, 0, 1)$

8. (a) $(4, -3, 1)$ (b) $(0, 2, -1)$
   (c) $(7, -8, 3)$ (d) $(1, 2, -1)$

9. (a) $(1, -1, 1, -1)$ (b) $(1, 2, 3, 4)$

Homework 13

1. (a) $\{(1, 0, 1)\}$ (b) $\emptyset$
   (c) $\{(3, 1, 0), (-1, 0, 1)\}$ (d) $\{(4, -5, 1)\}$

2. (a) $\{(-\frac{1}{4}, -\frac{1}{4}, 1, 0), (0, -1, 0, 1)\}$
   (b) $\{(4, 1, 0, 0), (-3, 0, 1, 0), (1, 0, 0, 1)\}$

3. (a) $\{(\frac{2}{3}, 1, 0), (-\frac{5}{3}, 0, 1)\}$
   (b) $\{(1, 1, 0), (0, 0, 1)\}$
   (c) $\{(2, -1, 4)\}$
   (d) $\{(1, 1, 0), (0, 1, 1)\}$

4. (a) $\{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0)\}$
   (b) $\{(1, 0, 1, 1), (0, 1, -1, 1)\}$
   (c) $\{(1, 1, 1, 1)\}$

5. (a) 3 (b) 6 (c) 6

6. (a) $(0, 1, 0)$ (b) $(0, 0, 1)$ (c) $(1, 0, 0)$

7. (a) $(0, 1, 0, 0), (0, 0, 1, 0)$
   (b) $(0, 0, 1, 0), (0, 0, 0, 1)$

8. $(1, 0, 0), (1, 0, 1)$

9. $(1, 1, 1, 1), (2, 2, 2, 0), (0, 0, 0, 3)$

Homework 14

1. (a) $\begin{bmatrix} \frac{13}{10} & \frac{-1}{2} \\ \frac{-2}{5} & 0 \end{bmatrix}, \begin{bmatrix} 0 & \frac{-5}{2} \\ -2 & \frac{-13}{2} \end{bmatrix}, \begin{bmatrix} \frac{-17}{10} \\ \frac{8}{5} \end{bmatrix}, \begin{bmatrix} -4 \\ -7 \end{bmatrix}$
   (b) $\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{-1}{11} & \frac{2}{11} \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \end{bmatrix}, \begin{bmatrix} \frac{-3}{11} \\ \frac{-13}{11} \end{bmatrix}$
   (c) $\begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ -1 & -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$
   (d) $\begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ \frac{-1}{11} & \frac{2}{11} \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} \frac{29}{11} \\ \frac{1}{11} \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

2. (a) $\begin{bmatrix} 3 & 2 & \frac{5}{2} \\ -2 & -3 & -\frac{1}{2} \\ 5 & 1 & 6 \end{bmatrix}, \begin{bmatrix} 9 \\ -9 \\ -5 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} -7 \\ 23 \\ 12 \end{bmatrix}$
   (b) $\frac{1}{12}\begin{bmatrix} 9 & 9 & 1 \\ -9 & -17 & -17 \\ 0 & 8 & 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{12}\begin{bmatrix} 19 \\ -43 \\ 16 \end{bmatrix}$
   (c) $\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -239 \\ 77 \\ 30 \end{bmatrix}$

3. (a) $\begin{bmatrix} -\frac{2}{9} & \frac{7}{9} \\ \frac{1}{3} & -\frac{1}{6} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{4} & \frac{7}{2} \\ \frac{3}{2} & 1 \end{bmatrix}$
   (c) $(1, -1)$ (d) $(-\frac{11}{4}, \frac{1}{2})$

4. (b) $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$
   (d) $\begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

5. (a) $\begin{bmatrix} 31 & 11 \\ 7 & 2 \end{bmatrix}$ (b) $\frac{1}{5}\begin{bmatrix} 2 & -11 \\ -7 & 2 \end{bmatrix}$

6. (a) $\{(1, 1, 0), (1, 0, 2), (0, 2, 1)\}$
   (b) $\frac{1}{5}\{(4, 1, -2), (1, -1, 2), (-2, 2, 1)\}$

7. $\{(1, 0, 0), (4, 4, 3), (3, 3, 2)\}$

Homework 15

1. (a) No (b) $(1, -3, 1)$
   (c) No (d) $(-26, 13, -7, 4)$

2. (a) $(1, 0) + t(3, 1)$
   (b) $(-2, 7, 0) + t(-1, -1, 1)$
   (c) $(-1, 0, 0, 0) + r(2, 1, 0, 0) + s(-1, 0, 1, 0) + t(-2, 0, 0, 1)$
   (d) $(\frac{6}{5}, \frac{7}{5}, 0, 0) + s(\frac{7}{5}, \frac{4}{5}, 1, 0) + t(\frac{1}{5}, -\frac{3}{5}, 0, 1)$

3. (a) $\{(16, 19, 1)\};\ \{(1, 0, -16), (0, 1, -19)\}$
   (b) $\{(0, 1, 0), (\frac{1}{2}, 0, 1)\};\ \{(1, 0, -\frac{1}{2})\}$
   (c) $\{(-1, -1, 1, 0), (\frac{2}{7}, -\frac{4}{7}, 0, 1)\};$
   $\{(1, 0, 1, -\frac{2}{7}), (0, 1, 1, \frac{4}{7})\}$

(d) $\{(-1,-1,1,0,0),(-2,-1,0,1,0),$
$(-1,-2,0,0,1)\};\{(1,0,1,2,1),(0,1,1,1,2)\}$

4. (a) $\{1,0,11,0,3),(0,1,3,0,0),(0,0,0,1,0)\}$
(b) $\{(1,-2,-1,-3),(-2,5,3,8),(0,0,1,1)\}$
(c) $\{(1,-2,5,0,3),(-2,5,-7,0,-6),$
$(-1,3,-2,1,-3)\}$

5. (a) $\{(1,4,5,2),(2,1,3,0)\}$
(b) $\{(1,4,5,6,9),(3,-2,1,4,-1)\}$

6. (a) $\{(1,1,-4,-3),(2,0,2,-2),(2,-1,3,2)\}$
(b) $\{(1,1,0,0),(0,0,1,1),(-2,0,2,2),$
$(0,-3,0,3)\}$

7. $\{\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_4\};\ \mathbf{v}_3=2\mathbf{v}_1-\mathbf{v}_2;$
$\mathbf{v}_5=-\mathbf{v}_1+3\mathbf{v}_2+2\mathbf{v}_4$

Homework 16

1. (a) 3, 0 (b) 2, 1 (c) 1, 2 (d) 3, 1

2. (a) 1, 3 (b) 3, 2 (c) 3, 2 (d) 3, 1

3. (a) 3 (b) 6 (c) 3 (d) 2

4. (a) 6 (b) 0 (c) 6 (d) 4

5. (a) 0 to 4 (b) 2 to 6 (c) 0 to 4 (d) 0 to 4

Homework 17

1. (a) $(2,-1)$ (b) $(2,-5,-3)$ (c) $(2,0)$
(d) $(0,1,3)$ (e) $\frac{1}{2}(7\sqrt{2},-\sqrt{2})$ (f) $(-\frac{1}{2},1)$
(g) $(4,-2,6)$ (h) $(-\frac{1}{2},2)$

2. (a) $\begin{bmatrix}1&0\\0&-1\end{bmatrix}$ (b) $\begin{bmatrix}0&0\\0&\frac{1}{2}\end{bmatrix}$
(c) $\dfrac{1}{2}\begin{bmatrix}3&3\sqrt{3}\\3\sqrt{3}&-3\end{bmatrix}$ (d) $\dfrac{1}{2}\begin{bmatrix}0&0\\1&-\sqrt{3}\end{bmatrix}$

3. $(5x+4y,x-4y);\ (3x+3y,6x-2y)$

4. $(4x+8y,-2x-4y-z,-x-2y+3z);$
$(2y,x+3y,17x+3y)$

5. (a) No
(b) $(\frac{1}{17}w_1+\frac{3}{17}w_2,-\frac{5}{17}w_1+\frac{2}{17}w_2)$
(c) $(\frac{1}{3}w_1-\frac{2}{3}w_2,\frac{1}{3}w_1+\frac{1}{3}w_2)$
(d) No

6. (a) No
(b) $(-40w_1+16w_2+9w_3,$
$13w_1-5w_2-3w_3,5w_1-2w_2-w_3)$
(c) $(w_1-2w_2+4w_3,-w_1+2w_2-3w_3,$
$-w_1+3w_2-5w_3)$
(d) No

7. (a) Yes (b) No (c) Yes (d) No

Homework 18

1. (a) -1 (b) 4 (c) 5 (d) 0

2. (a) $(\lambda-3)(\lambda-7)(\lambda-1)=0$
(b) $(\lambda-9)(\lambda+1)(\lambda-3)(\lambda-7)=0$

3. (a) $\{(1,1)\};\{(-2,1)\}$ (b) $\emptyset$ (c) $\{(1,0),(0,1)\}$
(d) $\{(1,0)\}$ (e) $\{(1,1)\};\{(1,-1)\}$ (f) $\{(1,0)\}$
(g) $\{(1,0),(0,1)\}$ (h) $\emptyset$

4. (a) $\{(0,1,0)\};\{(-1,2,2)\};\{(-1,1,1)\}$
(b) $\{(2,0,1),(0,1,0)\};\{(1,0,-2)\}$
(c) $\{(1,0,1)\};\{(8,0,1)\}$
(d) $\{(1,-1,0),(1,0,-1)\};\{(1,1,1)\}$
(e) $\{(0,1,0),(1,0,1)\}$
(f) $\{(1,1,2)\};\{(1,1,0),(1,0,-1)\}$

5. (a) $\lambda=5,\{(1,1)\};\ \lambda=-1,\{(-2,1)\}$
(b) $\lambda=1,\{(1,1,0),(1,0,1)\};\ \lambda=2,\{(1,1,-1)\}$

6. (b) $\lambda=-\omega^2$

7. $\begin{bmatrix}-\frac{1}{2}&-\frac{1}{2}&1\\-\frac{1}{2}&-\frac{1}{2}&-1\\0&0&1\end{bmatrix}$

Homework 19

1. (a) $\begin{bmatrix}1&0\\3&1\end{bmatrix}$ (b) $\begin{bmatrix}\frac{4}{5}&\frac{3}{4}\\1&1\end{bmatrix}$
(c) $\begin{bmatrix}1&0&-2\\0&1&0\\0&0&1\end{bmatrix}$ (d) $\begin{bmatrix}0&1&0\\1&0&1\\-1&0&1\end{bmatrix}$

2. (a) $\begin{bmatrix}24234&-34815\\-23210&35839\end{bmatrix}$
(b) $\begin{bmatrix}1&0\\-1023&1024\end{bmatrix}$
(c) $\begin{bmatrix}1&0&0\\0&512&512\\0&512&512\end{bmatrix}$

3. $\begin{bmatrix}-1&10237&-2047\\0&1&0\\0&10245&-2048\end{bmatrix}$

4. (a) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ (b) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
(c) $\begin{bmatrix}1&-2&8\\0&-1&0\\0&0&-1\end{bmatrix}$ (d) $\begin{bmatrix}1&-2&8\\0&-1&0\\0&0&-1\end{bmatrix}$

5. $\frac{1}{6}\begin{bmatrix} 1+a+b & 2-b & 1-a+b \\ 2-b & 4+b & 2-b \\ 1-a+b & 2-b & 1+a+b \end{bmatrix}$
where $a = 3^{n+1}$ and $b = 2^{2n+1}$

Homework 20

1. (a) 12 (b) -18 (c) -9
(d) $\sqrt{30}$ (e) $\sqrt{11}$ (f) $\sqrt{203}$

2. (a) $\frac{23}{2}$ (b) -30 (c) -15
(d) $\frac{7}{\sqrt{2}}$ (e) $\sqrt{7}$ (f) $\sqrt{157}$

3. (a) -101 (b) 30 (c) 3 (d) $4\sqrt{14}$

4. (a) -24 (b) -42

5. (a) $\sqrt{65}$; $12\sqrt{5}$ (b) $\sqrt{58}$; $3\sqrt{13}$

6. (a) 3 (b) 56

7. (a) $\sqrt{93}$; $3\sqrt{11}$ (b) $\sqrt{39}$; $\sqrt{43}$

8. (a) -29 (b) -15

9. (a) $\sqrt{14}$; $\sqrt{137}$ (b) $\sqrt{30}$; $\sqrt{89}$

Homework 21

1. (a) $-\frac{1}{\sqrt{2}}$ (b) 0 (c) $-\frac{1}{\sqrt{2}}$

2. (a) 0 (b) 0

3. (a) $\frac{19\sqrt{7}}{70}$ (b) 0

4. (a) $\frac{4}{3}$ (b) 0

5. (a) $1 \leq \sqrt{132}$
(b) $20 \leq \sqrt{798}$
(c) $6 \leq 2\sqrt{30}$

6. (a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{3}}$; $\frac{1}{\sqrt{5}}$ (c) $\frac{\sqrt{15}}{4}$ (d) $\frac{1}{\sqrt{30}}$

7. (a) 0 (b) $\frac{4}{\sqrt{15}}$; $\frac{4}{\sqrt{6}}$ (c) 0 (d) $\frac{2\sqrt{14}}{\sqrt{15}}$

Homework 22

1. (a) $-\frac{11}{5}\mathbf{v}_1 - \frac{2}{5}\mathbf{v}_2 + 2\mathbf{v}_3$
(b) $-\frac{37}{5}\mathbf{v}_1 - \frac{9}{5}\mathbf{v}_2 + 4\mathbf{v}_3$

2. $0\mathbf{v}_1 - \frac{2}{3}\mathbf{v}_2 + \frac{1}{3}\mathbf{v}_3$

3. $\frac{1}{7}\mathbf{v}_1 + \frac{5}{21}\mathbf{v}_2 + \frac{1}{3}\mathbf{v}_3 + \mathbf{v}_4$

4. (a) $\frac{7}{3}p_1 + p_2 + \frac{8}{3}p_3$
(b) $-\frac{1}{3}p_1 + 0p_2 - \frac{14}{3}p_3$
(c) $4p_1 + 3p_2 + 0p_3$

5. (a) $\{(\frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}}), (\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})\}$
(b) $\{(1,0),(0,-1)\}$

6. (a) $\{(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}})\}$
(b) $\{(1,0,0), (0, \frac{7}{\sqrt{53}}, \frac{-2}{\sqrt{53}}), (0, \frac{30}{\sqrt{11925}}, \frac{105}{\sqrt{11925}})\}$

7. $\{(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0), (\frac{5}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{2}{\sqrt{30}}, 0),$
$(\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}, \frac{-2}{\sqrt{10}}, \frac{-2}{\sqrt{10}}), (\frac{1}{\sqrt{15}}, \frac{1}{\sqrt{15}}, \frac{-2}{\sqrt{15}}, \frac{3}{\sqrt{15}})\}$

8. $\{(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}), (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}), (\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, 0)\}$

9. $\{1, -\sqrt{3} + 2x\sqrt{3}, \sqrt{5} - 6x\sqrt{5} + 6x^2\sqrt{5}\}$

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